Supplemental material for "Bartlett and Bartlett-type corrections for censored data from a Weibull distribution"

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A A second application: Smokers data set

This data set is related to a clinical trial on the effectiveness of triple-combination pharmacotherapy for tobacco dependence treatment conducted by the Cancer Institute of New Jersey and Robert Wood Johnson Foundation. 127 smokers 18 years or older with predefined medical illnesses were recruited from the local community. The outcome were the `time` (in days) to first relapse (return to smoking). The study lasted 182 days (26 weeks). Therefore, the times are subject to a censoring type I (32% of times were censored). We only consider the 113 patients where such observed time was positive (non-zero). Other measures were assigned randomly treatment group with levels combination or patch only (`grp`), age in years at time of randomization (`age`), `gender` (male or female), `race` (white or non-white), `employment` (full-time or non-full-time), number of years the patient had been a smoker (`yearssmoking`), `levelsmoking` (heavy or light), the number of prior attempts to quit smoking (`priorattempts`) and the longest period of time, in days, that the patient has previously gone without smoking (`longestnosmoke`). We consider that $\texttt{time}_i \sim \mathrm{WE}(\theta_i, \sigma)$, where $\log\theta_i = \mathbf{X}_i^\top \boldsymbol{\beta}$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_{10})^\top$ and

$$\mathbf{X}_i^\top = (\texttt{grp}_i, \texttt{age}_i, \texttt{gender}_i, 1, \texttt{race}_i, \texttt{employment}_i, \texttt{yearssmoking}_i, \texttt{levelsmoking}_i, \\ \texttt{priorattempts}_i, \texttt{longestnosmoke}_i)$$

We test if the covariates `grp`,`age` and `gender` were significative to explain the `time`, i.e., we test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus $H_1 : \beta_j \neq 0$, for at least one $1 \leq j \leq 3$. For this particular problem $p = 10$ and $q = 3$. In order to illustrate the performance of the corrections, we consider 1,000 subsamples of size $n = 30$ each (the original size $n = 113$ is too large, so a correction might be unnecessary). In each subsample, σ was estimated via jackknife method. A summary for traditional and corrected versions of SLR, SR and ST test are presented in Table 1 considering levels of significance 1%, 5% and 10%. Note that the rate of subsamples where the null hypothesis is not rejected increase with the correction under the different levels of significance. In practical terms, this imply that the uncorrected SLR changed the decision at least in 6.3%, 10.4% and 10.7% of the subsamples for levels of significance 1%, 5% and 10% in comparison with the corrected SLR; the uncorrected SR changed the decision in 0.7%, 3.7% and 6.1% less of the subsamples for levels of significance 1%, 5% and 10% in comparison with the corrected SR; and the uncorrected ST changed the decision in 13.5%, 23.2% and 23.0% less of the subsamples for levels of significance 1%, 5% and 10% in comparison with the corrected ST. We also present the distribution of such statistics in Figure 1. As expected, in all the cases the correction provides a distribution closer to the $\chi^2_{(3)}$ model.

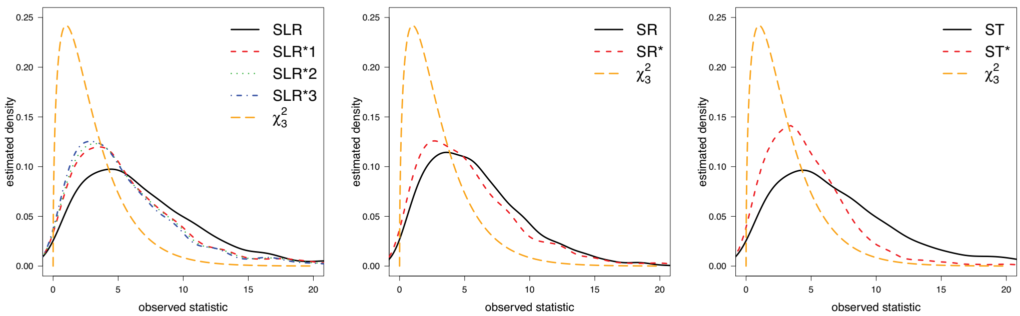

Figure 1: *Estimated density function for the traditional and corrected statistics in 1,000 subsamples of size* $n = 30$ *in smokers data set.*

Table 1: *Percentage of times where H_0 is not rejected in 1,000 subsamples of size $n = 30$ in smokers data set.*

Significance	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1%	0.837	0.912	0.825	0.900	0.900	0.901	0.919	0.960
5%	0.644	0.750	0.630	0.748	0.758	0.775	0.787	0.862
10%	0.531	0.620	0.522	0.638	0.650	0.670	0.681	0.752

B Additional Tables for simulation studies

B.1 Assessing the type I error

Simulated rejection rates for $H_0 : \beta_q = 0_q$.

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	10%	20	3	1	6.1	5.3	6.2	5.2	5.1	5.1	5.3	4.9
				2	5.9	5.6	6.3	4.8	4.8	4.7	5.4	4.8
			5	1	6.5	5.0	6.5	5.0	5.0	4.9	4.9	4.0
				2	6.7	5.0	6.9	4.8	4.7	4.6	5.1	4.2
				3	7.0	5.7	7.4	4.9	4.8	4.7	5.6	4.9
			7	1	6.7	4.8	6.7	5.1	4.9	4.8	4.4	3.2
				3	7.3	4.5	7.4	4.3	4.2	3.9	4.6	3.5
				5	7.6	5.7	8.4	4.4	4.3	4.0	4.8	4.4
		30	3	1	5.7	4.9	5.8	4.9	4.9	4.9	4.9	4.8
				2	5.9	5.5	6.2	5.1	5.1	5.1	5.5	5.1
			5	1	6.2	5.2	6.3	5.2	5.2	5.1	5.1	4.7
				2	6.3	5.1	6.5	5.0	5.0	4.9	5.1	4.8
				3	6.3	5.4	6.8	5.0	4.9	4.9	5.3	4.9
			7	1	6.1	4.8	6.2	4.8	4.7	4.6	4.6	3.7
				3	6.8	5.1	7.1	4.8	4.7	4.6	5.0	4.3
				5	7.2	5.7	7.8	4.9	4.9	4.7	5.3	5.1
		40	3	1	5.3	5.2	5.4	4.8	4.8	4.8	5.2	4.8
				2	5.7	5.3	6.0	5.0	4.9	4.9	5.2	5.0
			5	1	6.1	5.4	6.2	5.3	5.2	5.2	5.4	5.0
				2	6.0	5.4	6.2	5.1	5.0	5.0	5.3	4.8
				3	6.1	5.3	6.4	4.9	4.8	4.8	5.2	4.9
			7	1	6.1	5.0	6.1	5.1	5.0	5.0	4.8	4.3
				3	6.7	5.3	7.0	5.1	5.0	5.0	5.2	4.9
				5	6.6	5.6	7.4	5.0	4.9	4.9	5.3	5.0

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	25%	20	3	1	7.0	6.1	7.1	5.7	5.6	5.5	5.6	5.4
				2	7.1	5.7	7.3	5.5	5.4	5.3	5.7	5.5
			5	1	8.0	6.5	8.0	5.9	5.7	5.6	5.4	4.6
				2	8.8	6.5	8.9	6.0	5.8	5.6	5.7	5.4
				3	9.1	6.0	9.6	5.9	5.8	5.6	5.8	5.8
			7	1	9.4	7.3	9.4	6.6	6.3	5.9	5.5	3.6
				3	10.8	6.7	11.4	6.1	5.7	5.3	5.9	4.9
				5	10.9	5.9	12.2	5.9	5.6	5.2	6.0	5.8
		30	3	1	7.0	6.2	7.0	5.8	5.8	5.7	5.9	5.7
				2	7.5	6.3	7.6	6.1	6.0	6.0	6.3	6.2
			5	1	7.5	6.5	7.7	6.0	6.0	5.9	5.8	5.4
				2	8.3	6.5	8.4	6.1	6.0	5.9	6.0	5.9
				3	8.7	6.7	9.2	6.3	6.2	6.1	6.5	6.3
			7	1	8.4	6.8	8.5	6.3	6.1	5.9	5.7	4.7
				3	10.4	7.5	10.7	6.8	6.6	6.4	6.6	6.3
				5	10.6	6.6	11.5	6.7	6.5	6.3	6.6	6.8
		40	3	1	6.0	5.5	6.1	5.1	5.1	5.1	5.3	5.0
				2	6.5	5.6	6.5	5.6	5.5	5.5	5.6	5.5
			5	1	6.6	5.7	6.7	5.3	5.2	5.2	5.1	4.9
				2	7.0	6.0	7.2	5.5	5.4	5.3	5.6	5.3
				3	7.2	5.7	7.4	5.5	5.4	5.3	5.5	5.4
			7	1	7.3	6.2	7.3	5.6	5.5	5.4	5.1	4.7
				3	7.9	6.1	8.2	5.4	5.3	5.1	5.4	5.1
				5	8.6	5.7	9.1	5.8	5.7	5.5	5.6	5.9
	50%	20	3	1	7.7	6.8	7.7	5.7	5.6	5.4	6.2	5.9
				2	8.0	6.1	7.9	5.8	5.7	5.6	6.2	6.0
			5	1	9.2	7.7	9.3	6.2	5.9	5.4	5.9	5.8
				2	10.4	7.7	10.7	6.6	6.2	5.8	6.4	6.2
				3	10.8	6.7	11.1	6.5	6.2	5.8	6.4	6.4
			7	1	11.0	9.4	11.2	7.1	6.5	5.8	6.2	5.6
				3	13.2	8.4	14.1	7.1	6.5	5.7	6.7	5.8
				5	14.7	6.2	15.9	7.5	6.9	6.3	6.5	6.5
		30	3	1	6.9	6.3	6.8	5.6	5.5	5.4	5.8	5.6
				2	7.0	5.6	6.8	5.4	5.3	5.2	5.7	5.4
			5	1	8.0	7.2	8.0	6.0	5.8	5.5	6.0	5.9
				2	8.4	6.5	8.3	5.7	5.5	5.3	5.7	5.8
				3	8.4	5.9	8.4	5.7	5.5	5.3	5.8	5.7
			7	1	8.8	7.5	8.8	6.0	5.7	5.2	5.7	5.4
				3	10.1	7.1	10.1	5.9	5.6	5.2	5.8	5.7
				5	10.7	5.9	10.8	6.1	5.8	5.4	6.1	6.2
		40	3	1	6.2	5.8	6.2	5.2	5.1	5.1	5.4	5.2
				2	6.7	5.6	6.4	5.3	5.3	5.2	5.6	5.3
			5	1	7.4	6.7	7.3	5.6	5.5	5.4	5.8	5.6
				2	7.1	6.0	7.1	5.1	5.0	4.9	5.4	5.2
				3	7.5	5.6	7.2	5.2	5.1	5.0	5.4	5.3
			7	1	7.9	7.1	7.9	5.6	5.4	5.2	5.7	5.3
				3	8.9	6.7	8.9	5.7	5.5	5.2	5.8	5.6
				5	9.0	5.6	8.9	5.8	5.6	5.4	5.7	5.8

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	10%	20	3	1	6.2	5.3	6.3	5.2	5.1	5.1	5.2	4.9
				2	6.4	5.8	6.7	5.3	5.2	5.2	5.6	5.3
			5	1	6.2	4.8	6.2	4.9	4.8	4.7	4.7	4.0
				2	6.6	5.0	6.7	4.7	4.7	4.6	5.0	4.3
				3	6.8	5.6	7.3	4.7	4.6	4.5	5.4	4.7
			7	1	7.2	5.1	7.1	5.2	5.1	4.9	4.6	3.3
				3	7.6	4.8	7.6	4.6	4.4	4.2	4.9	3.9
				5	7.9	5.9	8.8	4.6	4.5	4.3	5.0	4.7
		30	3	1	5.3	4.8	5.4	4.6	4.6	4.6	4.7	4.6
				2	5.9	5.4	6.2	5.2	5.1	5.1	5.3	5.1
			5	1	6.3	5.2	6.3	5.2	5.1	5.0	5.2	4.6
				2	6.2	5.1	6.4	4.8	4.8	4.7	5.2	4.6
				3	6.3	5.5	6.8	4.8	4.7	4.7	5.4	4.7
			7	1	6.7	5.1	6.7	5.4	5.3	5.2	5.0	4.3
				3	6.9	5.1	7.0	4.7	4.7	4.5	5.2	4.4
				5	7.1	5.6	7.9	5.0	4.9	4.8	5.2	4.8
		40	3	1	5.8	5.4	5.9	5.2	5.1	5.1	5.4	5.1
				2	5.6	5.4	5.9	5.0	5.0	5.0	5.4	5.0
			5	1	6.0	5.1	6.1	5.2	5.1	5.1	5.1	4.9
				2	6.2	5.5	6.4	5.3	5.2	5.2	5.6	5.1
				3	5.9	5.4	6.4	4.8	4.8	4.8	5.3	4.8
			7	1	6.2	5.0	6.2	5.1	5.0	4.9	5.0	4.4
				3	6.3	5.0	6.5	4.7	4.7	4.6	5.1	4.5
				5	6.6	5.5	7.2	4.9	4.8	4.7	5.2	4.8
	25%	20	3	1	7.1	6.2	7.1	5.6	5.6	5.5	5.7	5.3
				2	7.4	5.7	7.7	5.5	5.5	5.3	5.7	5.7
			5	1	7.9	6.5	8.0	5.8	5.7	5.4	5.2	4.6
				2	8.8	6.5	9.1	5.9	5.7	5.4	5.9	5.2
				3	9.1	6.0	9.7	5.9	5.8	5.6	5.8	5.7
			7	1	9.1	7.2	9.1	6.2	6.0	5.7	5.4	3.6
				3	11.3	7.3	11.9	6.6	6.3	5.9	6.2	5.3
				5	10.8	5.6	12.0	6.0	5.6	5.2	5.7	5.7
		30	3	1	6.9	6.2	6.9	5.7	5.7	5.6	5.8	5.6
				2	7.3	6.2	7.6	5.9	5.9	5.8	6.2	6.0
			5	1	7.6	6.5	7.7	6.0	6.0	5.9	5.8	5.4
				2	8.6	6.7	8.7	6.2	6.1	6.0	6.1	6.0
				3	8.6	6.7	9.1	6.2	6.1	6.0	6.5	6.2
			7	1	8.2	6.8	8.3	6.0	5.9	5.8	5.7	4.7
				3	10.1	7.5	10.4	6.6	6.4	6.2	6.6	6.0
				5	10.6	6.6	11.5	6.7	6.4	6.1	6.5	6.7
		40	3	1	6.4	5.8	6.5	5.4	5.4	5.4	5.5	5.3
				2	6.3	5.6	6.4	5.3	5.3	5.3	5.5	5.3
			5	1	6.8	6.0	6.9	5.6	5.5	5.5	5.4	5.1
				2	6.8	5.7	6.9	5.2	5.1	5.0	5.3	5.0
				3	7.2	5.5	7.4	5.2	5.2	5.1	5.3	5.2
			7	1	7.3	6.2	7.4	5.7	5.6	5.5	5.3	4.8
				3	7.8	5.8	8.1	5.4	5.2	5.1	5.1	5.1
				5	8.1	5.7	8.7	5.4	5.3	5.2	5.6	5.4

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = \mathbf{0}_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	50%	20	3	1	7.6	6.6	7.5	5.7	5.5	5.3	5.9	5.7
				2	7.9	6.1	7.8	5.6	5.5	5.3	6.1	5.9
			5	1	9.6	8.2	9.6	6.5	6.1	5.7	6.1	5.9
				2	10.3	7.5	10.6	6.6	6.2	5.8	6.3	6.3
				3	10.6	6.5	10.8	6.5	6.1	5.8	6.4	6.4
			7	1	10.8	9.1	10.9	6.8	6.4	5.7	6.2	5.5
				3	14.2	8.9	15.0	7.6	7.0	6.3	7.0	6.1
				5	14.5	5.9	15.9	7.3	6.8	6.1	6.3	6.7
		30	3	1	6.3	5.7	6.3	4.9	4.8	4.8	5.3	5.0
				2	7.2	5.7	7.0	5.5	5.4	5.3	5.7	5.5
			5	1	7.6	6.7	7.6	5.6	5.4	5.2	5.6	5.4
				2	8.4	6.8	8.2	5.9	5.7	5.5	6.0	6.0
				3	8.6	6.1	8.7	5.9	5.7	5.5	5.8	5.9
			7	1	8.6	7.5	8.6	5.5	5.3	4.9	5.5	5.1
				3	10.5	7.6	10.6	6.4	6.0	5.6	6.2	6.2
				5	11.0	5.9	11.2	6.5	6.2	5.9	6.1	6.6
		40	3	1	6.3	5.8	6.2	5.3	5.2	5.2	5.5	5.3
				2	6.4	5.6	6.3	5.2	5.2	5.2	5.6	5.2
			5	1	6.7	6.0	6.7	5.0	4.9	4.8	5.0	5.1
				2	7.3	6.2	7.2	5.3	5.2	5.1	5.6	5.4
				3	8.0	6.2	7.8	5.8	5.7	5.6	6.0	5.9
			7	1	7.9	7.3	7.9	5.8	5.6	5.4	5.8	5.6
				3	8.8	6.8	8.7	5.7	5.5	5.2	5.8	5.6
				5	9.2	5.6	9.2	5.8	5.6	5.4	5.7	5.9
3	10%	20	3	1	6.2	5.5	6.3	5.2	5.2	5.1	5.4	4.9
				2	6.2	5.7	6.6	5.1	5.0	5.0	5.5	5.1
			5	1	6.4	4.9	6.5	5.0	4.9	4.8	4.8	4.0
				2	6.8	5.3	7.1	5.0	4.9	4.8	5.3	4.5
				3	6.7	5.6	7.4	4.8	4.7	4.6	5.4	4.6
			7	1	7.1	4.9	7.0	5.2	5.1	4.9	4.4	3.2
				3	7.6	5.0	7.7	4.7	4.5	4.3	5.2	3.9
				5	7.7	5.7	8.7	4.5	4.3	4.2	4.8	4.7
		30	3	1	5.9	5.4	6.1	5.2	5.2	5.2	5.4	5.1
				2	6.0	5.6	6.3	5.3	5.3	5.2	5.5	5.3
			5	1	6.1	5.1	6.3	5.1	5.1	5.0	5.1	4.6
				2	6.2	5.3	6.5	5.0	5.0	4.9	5.2	4.7
				3	6.3	5.5	6.7	4.9	4.8	4.8	5.4	4.8
			7	1	6.6	5.1	6.6	5.1	5.0	4.9	4.9	4.0
				3	6.7	5.1	7.0	4.7	4.6	4.5	5.2	4.3
				5	7.2	5.9	8.2	5.0	4.9	4.8	5.5	5.1
		40	3	1	5.6	5.1	5.8	5.1	5.1	5.1	5.1	5.1
				2	5.4	5.2	5.8	4.9	4.9	4.8	5.1	4.8
			5	1	5.8	4.9	5.9	5.0	4.9	4.9	4.9	4.6
				2	6.1	5.4	6.5	5.0	5.0	4.9	5.5	4.9
				3	6.2	5.6	6.6	5.1	5.1	5.0	5.5	5.1
			7	1	6.0	5.0	6.2	5.1	5.0	5.0	4.9	4.4
				3	6.6	5.1	6.9	4.9	4.8	4.8	5.1	4.7
				5	6.5	5.6	7.2	4.8	4.8	4.7	5.3	4.8

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
3	25%	20	3	1	7.2	6.2	7.4	5.8	5.7	5.6	5.8	5.5
				2	7.7	6.0	8.0	6.0	5.9	5.8	6.0	5.9
			5	1	8.4	6.7	8.4	6.2	6.1	5.9	5.6	4.8
				2	8.7	6.5	9.0	6.1	5.9	5.6	5.7	5.3
				3	9.2	6.0	9.8	6.0	5.9	5.7	5.8	5.8
			7	1	9.0	6.9	8.9	6.2	6.0	5.8	5.2	3.5
				3	11.6	7.0	11.9	6.5	6.2	5.8	6.1	5.3
				5	11.6	6.1	12.9	6.4	6.0	5.7	6.2	6.3
		30	3	1	6.9	6.1	6.9	5.8	5.7	5.6	5.8	5.6
				2	7.4	6.3	7.5	6.1	6.1	6.0	6.2	6.0
			5	1	7.8	6.7	7.8	6.2	6.1	6.0	5.8	5.5
				2	8.6	6.8	8.8	6.3	6.2	6.1	6.4	6.0
				3	8.7	6.4	9.0	6.2	6.1	6.0	6.2	6.2
			7	1	8.3	7.0	8.4	6.4	6.2	6.0	5.7	4.9
				3	10.4	7.4	10.9	6.9	6.6	6.3	6.6	6.1
				5	10.3	6.4	11.2	6.4	6.2	6.0	6.4	6.6
		40	3	1	6.4	5.8	6.4	5.5	5.5	5.5	5.6	5.4
				2	6.6	5.7	6.8	5.6	5.6	5.5	5.7	5.7
			5	1	6.7	5.9	6.7	5.5	5.4	5.4	5.3	5.1
				2	7.1	5.9	7.3	5.5	5.5	5.4	5.5	5.3
				3	7.0	5.4	7.2	5.3	5.2	5.2	5.3	5.3
			7	1	7.3	6.2	7.4	5.6	5.5	5.4	5.2	4.7
				3	8.4	6.3	8.6	5.9	5.7	5.6	5.6	5.5
				5	8.2	5.6	8.5	5.6	5.5	5.4	5.5	5.6
	50%	20	3	1	7.6	6.5	7.6	5.8	5.6	5.5	5.8	5.7
				2	8.5	6.3	8.5	6.1	6.0	5.9	6.3	6.3
			5	1	9.4	8.1	9.4	6.4	6.1	5.7	6.2	5.7
				2	10.4	7.6	10.7	6.4	6.1	5.6	6.4	6.4
				3	10.6	6.4	11.0	6.3	6.1	5.7	6.1	6.4
			7	1	11.0	9.1	11.1	6.8	6.2	5.4	6.2	5.8
				3	14.4	9.0	15.0	7.8	7.1	6.3	7.0	6.4
				5	14.7	6.2	16.0	7.6	7.0	6.3	6.6	6.9
		30	3	1	6.4	5.8	6.4	5.0	4.9	4.8	5.3	5.1
				2	7.2	5.8	7.1	5.8	5.7	5.6	5.8	5.8
			5	1	7.6	6.9	7.6	5.7	5.5	5.3	5.6	5.5
				2	8.6	6.8	8.6	5.9	5.7	5.5	6.0	5.9
				3	8.3	5.9	8.3	5.6	5.4	5.2	5.7	5.5
			7	1	9.1	7.9	9.1	6.1	5.8	5.5	5.9	5.5
				3	10.4	7.4	10.6	6.2	5.8	5.4	6.1	6.0
				5	10.7	5.7	10.9	6.4	6.1	5.7	5.9	6.2
		40	3	1	6.4	5.8	6.4	5.3	5.3	5.3	5.4	5.4
				2	6.7	5.6	6.5	5.5	5.4	5.4	5.6	5.5
			5	1	7.1	6.4	7.0	5.4	5.2	5.1	5.5	5.4
				2	7.5	6.3	7.4	5.5	5.4	5.2	5.7	5.5
				3	7.9	6.0	7.8	5.7	5.6	5.4	5.8	5.9
			7	1	8.0	7.2	7.9	5.7	5.6	5.3	5.6	5.4
				3	8.6	6.5	8.7	5.6	5.4	5.2	5.6	5.5
				5	9.2	5.6	9.1	5.8	5.6	5.4	5.7	5.8

B.2 Assessing the power of the tests

Simulated rejection rates for $H_0 : \beta_q = 0_q$.

				ψ					
σ	C	q	statistic	0.05	0.10	0.25	0.50	1.00	2.00
0.5	10%	1	SLR	8.3	12.6	41.4	86.5	99.7	100.0
			SR	7.2	11.7	41.5	86.2	99.7	100.0
			ST	8.4	12.8	41.6	86.5	99.8	100.0
			SLR*1	9.0	13.2	40.7	85.6	99.7	100.0
			SLR*2	9.0	13.2	40.7	85.5	99.7	100.0
			SLR*3	8.9	13.2	40.6	85.5	99.7	100.0
			SR*	7.2	11.7	41.5	86.2	99.7	100.0
			ST*	8.2	12.9	44.2	88.3	98.4	100.0
		3	SLR	10.0	20.6	76.1	99.2	100.0	100.0
			SR	9.8	22.7	79.1	99.3	100.0	100.0
			ST	10.7	21.7	76.5	99.3	100.0	100.0
			SLR*1	10.7	21.3	75.6	99.2	100.0	100.0
			SLR*2	10.7	21.3	75.6	99.2	100.0	100.0
			SLR*3	10.7	21.3	75.5	99.2	100.0	100.0
			SR*	9.8	22.7	79.1	99.3	100.0	100.0
			ST*	10.5	22.3	79.0	98.7	100.0	100.0
	25%	1	SLR	9.3	13.3	39.3	82.6	99.4	100.0
			SR	7.9	12.6	41.2	83.5	99.4	100.0
			ST	9.3	13.4	39.6	82.7	99.4	100.0
			SLR*1	10.0	14.0	38.7	81.7	99.4	100.0
			SLR*2	9.9	13.9	38.7	81.6	99.4	100.0
			SLR*3	9.9	13.9	38.6	81.6	99.4	100.0
			SR*	7.9	12.6	41.2	83.5	99.4	100.0
			ST*	9.4	14.1	42.2	84.1	95.0	100.0
		3	SLR	11.9	21.6	71.4	98.4	100.0	100.0
			SR	10.1	23.4	76.8	98.8	100.0	100.0
			ST	12.7	22.8	72.2	98.5	100.0	100.0
			SLR*1	12.9	22.4	70.9	98.4	100.0	100.0
			SLR*2	12.9	22.4	70.9	98.4	100.0	100.0
			SLR*3	12.9	22.3	70.9	98.4	100.0	100.0
			SR*	10.1	23.4	76.8	98.8	100.0	100.0
			ST*	13.0	24.2	75.1	95.9	100.0	100.0

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

				ψ					
σ	C	q	statistic	0.05	0.10	0.25	0.50	1.00	2.00
0.5	50%	1	SLR	10.4	13.1	32.8	72.7	97.6	100.0
			SR	9.1	12.8	35.0	75.6	97.8	100.0
			ST	10.5	13.3	33.2	72.9	97.6	100.0
			SLR*1	10.3	13.3	32.7	72.7	97.6	99.9
			SLR*2	10.3	13.2	32.7	72.6	97.6	100.0
			SLR*3	10.2	13.1	32.5	72.6	97.6	100.0
			SR*	9.1	12.8	35.0	75.6	97.8	100.0
			ST*	11.5	14.3	34.6	71.0	92.0	100.0
		3	SLR	13.1	19.6	61.4	96.2	100.0	100.0
			SR	9.8	19.2	68.2	97.3	100.0	100.0
			ST	13.5	20.6	62.9	96.4	100.0	100.0
			SLR*1	13.2	19.7	61.3	96.3	100.0	100.0
			SLR*2	13.1	19.7	61.3	96.3	100.0	100.0
			SLR*3	13.1	19.7	61.2	96.2	100.0	100.0
			SR*	9.8	19.2	68.2	97.3	100.0	100.0
			ST*	16.0	23.9	65.5	88.1	99.5	100.0
1	10%	1	SLR	6.9	8.0	16.5	42.0	86.9	99.7
			SR	5.4	6.6	15.6	41.9	86.3	99.6
			ST	6.9	8.1	16.7	41.9	86.9	99.7
			SLR*1	7.7	8.7	17.0	41.3	86.1	99.7
			SLR*2	7.7	8.7	16.9	41.2	86.0	99.7
			SLR*3	7.6	8.7	16.9	41.2	85.9	99.7
			SR*	5.4	6.6	15.6	41.9	86.3	99.6
			ST*	6.7	7.8	17.3	44.7	88.8	98.5
		3	SLR	7.5	10.0	28.9	75.9	99.3	100.0
			SR	6.4	9.8	33.0	79.1	99.4	100.0
			ST	8.1	10.8	30.1	76.5	99.3	100.0
			SLR*1	8.2	10.9	29.3	75.5	99.3	100.0
			SLR*2	8.2	10.9	29.3	75.5	99.3	100.0
			SLR*3	8.1	10.9	29.2	75.5	99.3	100.0
			SR*	6.4	9.8	33.0	79.1	99.4	100.0
			ST*	7.8	10.6	31.4	78.8	98.5	98.9
	25%	1	SLR	8.7	9.1	16.5	39.1	82.6	99.3
			SR	7.2	8.1	16.3	40.8	83.5	99.3
			ST	8.8	9.2	16.8	39.3	82.6	99.3
			SLR*1	9.5	9.9	16.9	38.5	81.6	99.2
			SLR*2	9.5	9.9	16.9	38.4	81.6	99.2
			SLR*3	9.4	9.8	16.8	38.4	81.5	99.2
			SR*	7.2	8.1	16.3	40.8	83.5	99.3
			ST*	8.7	9.3	17.4	41.8	84.1	94.8

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

				ψ					
σ	C	q	statistic	0.05	0.10	0.25	0.50	1.00	2.00
1	25%	3	SLR	10.0	11.7	28.8	71.5	98.5	100.0
			SR	7.1	10.0	32.3	76.9	98.9	100.0
			ST	10.6	12.5	30.3	72.6	98.5	100.0
			SLR*1	11.0	12.7	29.3	71.1	98.4	100.0
			SLR*2	11.0	12.6	29.2	71.1	98.4	100.0
			SLR*3	11.0	12.6	29.2	71.1	98.4	100.0
			SR*	7.1	10.0	32.3	76.9	98.9	100.0
			ST*	10.5	12.7	32.3	75.4	96.0	100.0
	50%	1	SLR	9.4	10.1	15.4	32.5	72.4	97.3
			SR	8.0	8.8	15.3	34.9	75.0	97.6
			ST	9.5	10.3	15.6	33.0	72.7	97.3
			SLR*1	9.5	10.1	15.5	32.8	72.5	97.3
			SLR*2	9.4	10.0	15.5	32.6	72.3	97.3
			SLR*3	9.4	10.0	15.4	32.5	72.2	97.3
			SR*	8.0	8.8	15.3	34.9	75.0	97.6
			ST*	10.4	11.2	16.8	34.9	70.5	91.6
		3	SLR	11.1	13.1	25.7	62.3	96.2	99.9
			SR	7.0	9.6	27.0	68.7	97.5	100.0
			ST	11.2	13.6	26.9	63.7	96.4	99.9
			SLR*1	11.1	13.2	25.7	62.3	96.2	99.9
			SLR*2	11.0	13.2	25.6	62.3	96.2	99.9
			SLR*3	11.0	13.1	25.6	62.3	96.2	99.9
			SR*	7.0	9.6	27.0	68.7	97.5	100.0
			ST*	13.6	16.1	30.5	66.2	88.3	98.4
3	10%	1	SLR	6.7	6.9	7.7	11.0	23.2	60.9
			SR	5.3	5.4	6.3	9.7	22.4	60.5
			ST	6.7	6.9	7.7	11.1	23.3	60.8
			SLR*1	7.4	7.8	8.4	11.7	23.4	59.7
			SLR*2	7.4	7.7	8.3	11.6	23.4	59.6
			SLR*3	7.3	7.7	8.3	11.6	23.3	59.5
			SR*	5.3	5.4	6.3	9.7	22.4	60.5
			ST*	6.4	6.6	7.5	11.2	24.7	64.0
		3	SLR	6.7	7.2	9.0	16.8	45.8	91.4
			SR	5.6	5.9	8.5	17.8	50.5	92.6
			ST	7.2	7.6	9.6	17.9	47.2	91.6
			SLR*1	7.6	7.9	9.8	17.4	45.9	91.1
			SLR*2	7.6	7.9	9.8	17.4	45.9	91.1
			SLR*3	7.6	7.9	9.8	17.4	45.9	91.1
			SR*	5.6	5.9	8.5	17.8	50.5	92.6
			ST*	6.9	7.3	9.2	17.9	49.3	92.8

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

				ψ					
σ	C	q	statistic	0.05	0.10	0.25	0.50	1.00	2.00
3	25%	1	SLR	8.0	8.0	9.3	11.6	22.8	55.9
			SR	6.7	6.6	7.9	11.0	23.4	57.4
			ST	8.0	8.0	9.4	11.6	23.0	56.1
			SLR*1	8.9	8.9	10.2	12.3	23.0	55.0
			SLR*2	8.8	8.8	10.2	12.2	22.9	54.9
			SLR*3	8.8	8.8	10.1	12.2	22.9	54.8
			SR*	6.7	6.6	7.9	11.0	23.4	57.4
			ST*	8.0	8.0	9.6	12.0	24.4	59.0
		3	SLR	9.6	9.1	11.1	18.1	43.6	88.3
			SR	6.4	6.4	8.8	18.2	49.1	90.9
			ST	10.2	9.8	11.9	19.2	44.9	88.7
			SLR*1	10.6	10.1	12.0	18.9	43.6	87.9
			SLR*2	10.6	10.1	12.0	18.8	43.6	87.9
			SLR*3	10.6	10.0	12.0	18.8	43.6	87.9
			SR*	6.4	6.4	8.8	18.2	49.1	90.9
			ST*	10.1	9.6	11.9	19.8	47.6	89.8
	50%	1	SLR	9.4	9.4	9.7	11.7	19.6	47.3
			SR	8.0	7.9	8.5	11.0	20.7	50.4
			ST	9.3	9.4	9.7	11.8	20.1	47.6
			SLR*1	9.4	9.5	9.8	11.8	19.7	47.5
			SLR*2	9.3	9.4	9.7	11.7	19.6	47.4
			SLR*3	9.3	9.4	9.7	11.6	19.6	47.3
			SR*	8.0	7.9	8.5	11.0	20.7	50.4
			ST*	10.4	10.1	10.6	12.9	21.4	49.0
		3	SLR	11.0	10.7	12.6	17.2	36.8	81.1
			SR	6.9	6.8	8.9	15.5	41.5	85.8
			ST	11.5	11.2	13.1	18.1	38.6	82.0
			SLR*1	11.1	10.9	12.6	17.2	36.8	81.1
			SLR*2	11.1	10.9	12.5	17.2	36.8	81.1
			SLR*3	11.1	10.9	12.5	17.2	36.8	81.0
			SR*	6.9	6.8	8.9	15.5	41.5	85.8
			ST*	13.5	13.2	15.6	20.9	42.2	81.5

B.3 Changing the assumption of σ known

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$.

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	10%	20	3	1	6.3	5.2	6.4	5.3	5.3	5.3	5.2	5.1
				2	6.1	5.2	6.7	4.9	4.9	4.8	5.2	4.8
			5	1	6.7	5.1	6.6	5.4	5.3	5.2	4.9	4.2
				2	7.1	5.1	7.2	5.1	5.0	4.9	5.0	4.5
				3	7.1	5.5	7.8	5.0	4.9	4.9	5.5	5.0
			7	1	6.8	4.7	6.7	5.2	5.1	4.9	4.4	3.1
				3	7.8	4.8	7.8	4.8	4.6	4.4	4.8	3.8
				5	8.2	5.7	9.2	5.0	4.8	4.6	5.6	5.0
		30	3	1	5.8	4.9	6.0	5.2	5.1	5.1	4.9	5.0
				2	6.0	5.4	6.3	5.2	5.2	5.2	5.3	5.1
			5	1	6.3	5.1	6.3	5.2	5.2	5.2	5.1	4.7
				2	6.2	5.0	6.5	4.9	4.9	4.8	5.0	4.6
				3	6.5	5.3	7.0	5.0	5.0	4.9	5.2	5.0
			7	1	6.1	4.6	6.2	4.9	4.9	4.8	4.4	3.7
				3	7.2	5.2	7.4	5.1	4.9	4.8	5.1	4.5
				5	7.5	5.5	8.3	5.2	5.1	5.0	5.3	5.3
		40	3	1	5.4	5.0	5.5	4.9	4.9	4.9	5.0	4.9
				2	5.6	5.0	5.9	4.9	4.9	4.8	5.0	5.0
			5	1	6.1	5.2	6.2	5.2	5.2	5.2	5.2	4.9
				2	6.0	5.2	6.3	5.1	5.0	5.0	5.1	4.9
				3	6.1	5.2	6.6	5.0	5.0	4.9	5.1	5.0
			7	1	6.2	5.0	6.2	5.1	5.1	5.1	4.9	4.4
				3	6.7	5.4	7.1	5.2	5.2	5.1	5.2	4.9
				5	6.8	5.6	7.7	5.0	5.0	4.9	5.4	5.2

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	25%	20	3	1	6.8	5.5	7.0	5.5	5.4	5.4	5.1	5.1
				2	6.9	4.8	7.3	5.4	5.3	5.3	4.9	5.4
			5	1	8.0	6.1	8.0	6.0	5.8	5.6	4.9	4.3
				2	8.9	6.0	9.2	6.1	5.9	5.7	5.3	5.3
				3	8.9	5.3	9.8	5.8	5.7	5.4	5.2	5.6
			7	1	9.3	7.1	9.3	6.5	6.3	6.0	5.1	3.2
				3	10.7	6.3	11.4	6.3	5.9	5.5	5.2	4.7
				5	11.3	5.0	12.9	6.3	5.9	5.5	5.2	5.9
		30	3	1	6.5	5.6	6.6	5.5	5.5	5.4	5.2	5.4
				2	6.6	5.3	6.9	5.5	5.4	5.4	5.3	5.5
			5	1	7.0	5.6	7.1	5.4	5.3	5.2	4.9	4.7
				2	7.4	5.5	7.5	5.4	5.4	5.2	5.0	5.1
				3	7.9	5.5	8.2	5.5	5.4	5.3	5.3	5.5
			7	1	7.9	6.3	8.0	5.9	5.7	5.5	5.2	4.2
				3	9.2	6.2	9.5	5.9	5.7	5.5	5.5	5.2
				5	9.2	5.2	9.9	5.7	5.5	5.3	5.2	5.7
		40	3	1	5.9	5.2	5.9	5.0	5.0	4.9	5.0	4.9
				2	6.4	5.3	6.6	5.3	5.3	5.2	5.3	5.3
			5	1	6.3	5.4	6.4	5.2	5.2	5.1	4.8	4.8
				2	6.9	5.5	7.0	5.3	5.3	5.2	5.1	5.2
				3	7.1	5.2	7.2	5.2	5.2	5.1	5.0	5.3
			7	1	7.1	5.9	7.1	5.4	5.3	5.2	5.0	4.5
				3	7.7	5.6	8.0	5.5	5.4	5.2	5.0	5.1
				5	8.2	5.0	8.6	5.7	5.6	5.4	5.0	5.7
	50%	20	3	1	7.4	6.0	7.4	5.6	5.4	5.3	5.3	5.7
				2	7.6	5.0	7.7	5.5	5.5	5.3	5.0	5.8
			5	1	9.1	7.1	9.1	6.1	5.8	5.4	5.2	5.4
				2	10.3	6.6	10.4	6.4	6.0	5.6	5.3	6.0
				3	10.3	5.2	10.8	6.2	6.0	5.6	5.0	6.2
			7	1	11.0	8.7	11.3	7.0	6.4	5.7	5.7	5.5
				3	12.9	7.0	13.9	7.0	6.4	5.7	5.2	5.5
				5	13.8	4.2	15.2	7.1	6.6	6.0	4.6	6.3
		30	3	1	6.5	5.5	6.4	5.1	5.0	4.9	5.0	5.1
				2	6.9	4.9	6.7	5.2	5.2	5.1	5.0	5.4
			5	1	7.3	6.2	7.4	5.4	5.2	5.0	5.0	5.3
				2	7.7	5.7	7.6	5.3	5.1	4.9	4.9	5.3
				3	8.4	5.3	8.3	5.6	5.5	5.3	5.1	5.7
			7	1	8.5	7.1	8.5	5.8	5.5	5.1	5.1	5.2
				3	9.9	5.9	10.0	5.7	5.4	5.1	4.7	5.5
				5	10.3	4.7	10.4	5.8	5.5	5.2	4.9	5.9
		40	3	1	6.2	5.4	6.2	5.2	5.1	5.1	5.1	5.2
				2	6.2	4.8	6.1	5.1	5.1	5.1	4.8	5.2
			5	1	6.8	6.1	6.8	5.3	5.2	5.0	5.1	5.3
				2	7.3	5.8	7.3	5.4	5.3	5.2	5.1	5.4
				3	7.5	5.3	7.3	5.4	5.4	5.3	5.1	5.6
			7	1	7.6	6.8	7.6	5.6	5.5	5.2	5.3	5.3
				3	8.8	5.9	8.7	5.7	5.4	5.2	5.0	5.6
				5	8.8	4.7	8.8	5.6	5.4	5.2	4.8	5.6

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	10%	20	3	1	6.3	5.3	6.4	5.3	5.3	5.2	5.2	5.0
				2	6.5	5.6	6.8	5.5	5.4	5.4	5.6	5.4
			5	1	6.3	4.6	6.4	5.0	5.0	4.9	4.5	4.0
				2	7.0	5.1	7.2	5.1	5.0	4.9	5.1	4.5
				3	6.8	5.5	7.5	4.8	4.7	4.6	5.5	4.8
			7	1	7.3	5.0	7.3	5.4	5.3	5.1	4.6	3.0
				3	8.2	5.3	8.4	5.1	4.9	4.7	5.3	4.1
				5	8.4	5.8	9.7	5.1	5.0	4.8	5.7	5.2
		30	3	1	5.3	4.6	5.3	4.6	4.6	4.6	4.6	4.5
				2	5.9	5.3	6.3	5.2	5.1	5.1	5.2	5.3
			5	1	6.3	5.0	6.3	5.2	5.2	5.1	4.9	4.7
				2	6.2	5.1	6.4	5.0	4.9	4.9	5.0	4.7
				3	6.5	5.4	7.0	5.0	5.0	4.9	5.3	5.0
			7	1	6.7	5.0	6.7	5.4	5.4	5.3	4.8	4.3
				3	7.1	5.1	7.3	4.9	4.8	4.7	5.1	4.4
				5	7.4	5.5	8.1	5.2	5.1	5.0	5.3	5.1
		40	3	1	5.9	5.3	6.0	5.4	5.4	5.4	5.3	5.2
				2	5.7	5.3	6.1	5.0	5.0	5.0	5.3	5.1
			5	1	6.0	5.1	6.2	5.3	5.2	5.2	5.1	4.8
				2	6.1	5.3	6.4	5.2	5.1	5.1	5.2	5.0
				3	6.1	5.4	6.6	5.0	5.0	4.9	5.3	5.0
			7	1	6.3	4.9	6.3	5.2	5.2	5.1	4.9	4.5
				3	6.3	5.0	6.6	4.8	4.8	4.7	4.9	4.7
				5	6.6	5.3	7.3	5.0	4.9	4.9	5.1	5.0
	25%	20	3	1	6.9	5.6	6.9	5.6	5.5	5.4	5.2	5.2
				2	7.1	5.1	7.6	5.5	5.4	5.4	5.1	5.5
			5	1	7.8	6.0	7.9	5.9	5.8	5.7	5.0	4.3
				2	8.9	6.0	9.1	6.0	5.8	5.5	5.3	5.2
				3	8.9	5.3	9.3	5.9	5.7	5.5	5.1	5.6
			7	1	9.0	6.7	9.0	6.3	6.0	5.8	5.0	3.3
				3	11.5	6.6	12.0	6.7	6.4	6.0	5.6	5.0
				5	11.0	4.8	12.3	6.0	5.8	5.4	4.9	6.0
		30	3	1	6.2	5.4	6.3	5.2	5.2	5.1	5.0	5.0
				2	6.4	5.1	6.7	5.3	5.2	5.2	5.1	5.3
			5	1	6.8	5.6	6.9	5.3	5.3	5.2	4.9	4.6
				2	7.4	5.5	7.5	5.4	5.3	5.2	5.0	5.1
				3	7.7	5.4	8.1	5.4	5.3	5.2	5.2	5.4
			7	1	7.6	6.0	7.6	5.8	5.7	5.5	4.9	4.1
				3	9.2	6.1	9.3	6.0	5.7	5.4	5.3	5.3
				5	9.3	5.0	9.9	5.7	5.5	5.3	5.0	5.8
		40	3	1	6.1	5.4	6.3	5.3	5.2	5.2	5.1	5.2
				2	6.1	5.0	6.3	5.2	5.2	5.2	4.9	5.2
			5	1	6.7	5.8	6.8	5.4	5.3	5.3	5.1	5.0
				2	6.6	5.4	6.8	5.1	5.0	5.0	4.9	4.9
				3	6.9	5.1	7.2	5.1	5.0	5.0	4.9	5.1
			7	1	7.4	6.0	7.4	5.7	5.6	5.5	5.1	4.7
				3	7.6	5.4	8.0	5.4	5.3	5.1	4.9	5.0
				5	8.1	5.2	8.6	5.5	5.4	5.2	5.1	5.4

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = \mathbf{0}_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	50%	20	3	1	7.2	5.8	7.3	5.4	5.2	5.0	5.0	5.4
				2	7.7	4.7	7.6	5.5	5.3	5.2	4.7	5.7
			5	1	8.4	6.8	8.5	5.7	5.5	5.1	5.0	5.1
				2	10.1	6.6	10.3	6.4	6.1	5.7	5.6	6.0
				3	10.1	4.9	10.4	6.0	5.7	5.3	4.6	6.2
			7	1	10.8	8.4	11.0	6.8	6.4	5.6	5.5	5.2
				3	13.5	7.1	14.4	7.2	6.7	6.0	5.4	5.8
				5	13.9	4.0	15.5	7.0	6.6	5.8	4.3	6.5
		30	3	1	6.9	5.6	6.7	5.3	5.2	5.1	5.1	5.3
				2	6.7	4.8	6.5	5.1	5.0	4.9	4.8	5.2
			5	1	7.7	6.3	7.6	5.5	5.3	5.1	5.0	5.3
				2	8.1	6.0	8.0	5.5	5.4	5.2	5.0	5.6
				3	8.3	5.2	8.2	5.6	5.4	5.3	5.0	5.6
			7	1	8.4	7.0	8.4	5.8	5.6	5.2	5.1	5.1
				3	10.2	6.4	10.2	6.1	5.7	5.4	5.2	5.9
				5	10.4	4.6	10.6	6.0	5.6	5.4	4.8	6.2
		40	3	1	6.1	5.6	6.1	5.1	5.1	5.0	5.2	5.1
				2	6.1	4.7	6.0	5.0	5.0	4.9	4.8	5.1
			5	1	6.8	6.2	6.8	5.3	5.1	5.0	5.2	5.2
				2	6.8	5.4	6.7	5.1	5.0	4.9	4.8	5.1
				3	7.4	5.2	7.1	5.4	5.3	5.1	5.0	5.4
			7	1	7.9	6.8	7.8	5.7	5.6	5.3	5.4	5.4
				3	8.9	6.3	8.8	5.6	5.4	5.2	5.4	5.6
				5	8.9	4.8	8.9	5.6	5.5	5.2	5.0	5.8
3	10%	20	3	1	6.1	5.1	6.2	5.2	5.1	5.1	5.1	4.9
				2	6.3	5.3	6.7	5.2	5.2	5.2	5.3	5.2
			5	1	6.6	4.8	6.6	5.2	5.1	5.0	4.6	4.0
				2	6.9	5.2	7.1	5.1	5.0	4.9	5.3	4.6
				3	7.1	5.5	7.7	4.9	4.8	4.7	5.6	4.8
			7	1	7.2	4.9	7.1	5.3	5.2	5.0	4.5	3.1
				3	8.0	5.3	8.2	5.0	4.8	4.6	5.3	4.1
				5	8.3	5.8	9.5	4.9	4.7	4.5	5.6	5.0
		30	3	1	5.8	5.2	5.9	5.2	5.2	5.2	5.2	5.0
				2	6.1	5.3	6.4	5.3	5.3	5.2	5.2	5.3
			5	1	6.0	5.1	6.2	5.2	5.1	5.1	5.1	4.6
				2	6.4	5.3	6.7	5.1	5.0	5.0	5.2	4.8
				3	6.4	5.2	6.8	5.0	4.9	4.9	5.1	5.0
			7	1	6.5	4.9	6.6	5.2	5.1	5.1	4.8	4.1
				3	7.1	5.1	7.4	5.0	4.9	4.8	5.1	4.5
				5	7.6	5.7	8.3	5.2	5.1	5.0	5.4	5.3
		40	3	1	5.6	5.0	5.7	5.1	5.1	5.1	4.9	5.0
				2	5.3	4.9	5.6	4.7	4.7	4.7	4.8	4.7
			5	1	5.8	4.9	5.9	5.0	4.9	4.9	4.9	4.7
				2	6.3	5.3	6.6	5.2	5.2	5.1	5.2	5.1
				3	6.2	5.4	6.7	5.1	5.1	5.1	5.3	5.2
			7	1	6.1	4.8	6.3	5.1	5.1	5.0	4.8	4.4
				3	6.8	5.1	7.1	5.1	5.0	4.9	5.0	4.7
				5	7.0	5.4	7.7	5.0	5.0	4.9	5.2	5.0

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
3	25%	20	3	1	6.8	5.5	7.0	5.5	5.5	5.4	5.1	5.2
				2	7.5	5.1	7.7	5.8	5.7	5.6	5.1	5.8
			5	1	8.1	6.3	8.1	6.0	5.9	5.7	5.1	4.5
				2	8.4	5.8	8.7	5.7	5.5	5.3	4.9	5.0
				3	9.1	5.3	9.6	5.9	5.8	5.5	5.1	5.7
			7	1	8.8	6.4	8.7	6.1	5.8	5.6	4.8	3.0
				3	11.2	6.5	11.8	6.7	6.3	5.9	5.5	5.0
				5	11.5	5.2	12.9	6.3	6.0	5.5	5.4	6.0
		30	3	1	6.4	5.2	6.4	5.3	5.3	5.2	4.9	5.1
				2	6.6	5.1	6.8	5.4	5.4	5.3	5.1	5.3
			5	1	7.1	5.7	7.2	5.5	5.4	5.3	4.9	4.8
				2	8.0	6.0	8.1	5.8	5.7	5.6	5.4	5.4
				3	7.7	5.3	8.1	5.4	5.3	5.2	5.1	5.4
			7	1	7.7	5.9	7.8	5.8	5.7	5.5	4.9	4.0
				3	9.1	6.0	9.6	5.9	5.7	5.5	5.2	5.3
				5	9.1	5.0	9.9	5.5	5.3	5.1	5.0	5.6
		40	3	1	6.3	5.5	6.4	5.5	5.5	5.4	5.2	5.4
				2	6.5	5.3	6.7	5.4	5.4	5.4	5.3	5.5
			5	1	6.7	5.7	6.7	5.5	5.4	5.4	5.1	5.0
				2	7.0	5.4	7.2	5.4	5.4	5.3	4.9	5.2
				3	6.8	5.1	7.0	5.2	5.1	5.1	5.0	5.2
			7	1	7.3	5.9	7.4	5.6	5.5	5.4	5.0	4.5
				3	8.0	6.0	8.4	5.6	5.5	5.3	5.2	5.3
				5	8.1	5.1	8.6	5.5	5.4	5.3	5.1	5.4
	50%	20	3	1	7.3	5.9	7.2	5.5	5.3	5.2	5.0	5.5
				2	7.9	4.9	7.9	5.5	5.3	5.2	5.0	5.7
			5	1	8.9	6.7	8.9	6.0	5.6	5.3	4.9	5.3
				2	10.0	6.4	10.0	6.1	5.8	5.4	5.3	5.8
				3	10.4	5.1	10.7	6.3	5.9	5.6	4.8	6.3
			7	1	10.3	8.2	10.6	6.6	6.1	5.4	5.3	5.2
				3	13.6	7.2	14.4	7.2	6.6	5.8	5.3	5.6
				5	13.9	4.1	15.4	7.2	6.5	5.9	4.4	6.4
		30	3	1	6.5	5.7	6.4	5.3	5.2	5.1	5.2	5.3
				2	6.9	5.0	6.8	5.4	5.3	5.2	5.1	5.5
			5	1	7.4	6.2	7.4	5.3	5.1	5.0	5.0	5.1
				2	7.9	5.9	8.0	5.6	5.4	5.2	5.1	5.5
				3	8.5	5.3	8.5	5.6	5.5	5.3	5.1	5.8
			7	1	8.8	7.3	8.8	5.9	5.7	5.3	5.3	5.2
				3	9.9	6.3	10.2	6.0	5.7	5.2	5.1	5.8
				5	10.2	4.7	10.5	6.1	5.7	5.4	4.8	6.2
		40	3	1	6.0	5.3	5.9	4.9	4.9	4.8	4.9	5.0
				2	6.1	4.8	5.8	4.9	4.8	4.8	4.8	4.9
			5	1	7.0	6.1	6.9	5.3	5.3	5.1	5.2	5.5
				2	7.3	5.7	7.1	5.3	5.3	5.1	5.1	5.4
				3	7.4	5.2	7.2	5.3	5.2	5.0	5.0	5.5
			7	1	7.5	6.7	7.6	5.4	5.2	5.0	5.1	5.1
				3	8.3	6.0	8.4	5.4	5.3	5.1	5.1	5.4
				5	8.7	4.7	8.6	5.5	5.3	5.1	4.8	5.6

B.4 Changing the assumption of censoring type I or II

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$.

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	10%	20	3	1	5.0	4.1	5.0	5.1	5.1	5.1	5.2	5.1
				2	4.7	4.8	4.9	4.9	4.9	4.9	4.1	4.8
			5	1	5.1	3.4	5.0	5.0	4.9	4.9	5.1	5.0
				2	4.7	3.7	4.8	4.7	4.7	4.7	5.2	4.7
				3	5.0	4.3	5.2	5.0	5.0	5.0	4.2	4.9
			7	1	5.6	3.1	5.4	5.0	4.9	4.8	5.1	5.0
				3	5.3	3.3	5.0	4.8	4.8	4.7	5.5	4.9
				5	5.1	5.2	5.3	4.8	4.8	4.8	1.6	5.0
		30	3	1	4.7	4.3	4.8	4.9	4.9	4.9	4.9	4.9
				2	4.5	4.9	4.9	4.7	4.7	4.7	4.5	4.7
			5	1	4.8	3.6	4.8	4.9	4.9	4.9	5.0	4.8
				2	5.0	4.3	5.1	5.3	5.3	5.3	5.5	5.2
				3	4.7	4.6	5.0	5.1	5.1	5.1	4.6	4.9
			7	1	5.2	3.4	5.2	5.2	5.2	5.2	5.4	5.2
				3	5.3	4.0	5.4	5.3	5.3	5.3	5.7	5.4
				5	4.7	5.3	5.1	4.9	4.9	4.9	2.8	4.8
		40	3	1	5.0	4.7	5.2	5.2	5.2	5.2	5.3	5.2
				2	4.9	5.0	5.2	5.2	5.2	5.2	4.8	5.1
			5	1	4.7	3.8	4.8	5.0	5.0	5.0	5.1	4.9
				2	4.7	4.1	5.0	5.0	5.0	5.0	5.0	5.0
				3	4.8	4.9	5.1	5.0	5.0	5.0	4.9	5.0
			7	1	4.8	3.4	4.8	5.0	5.0	5.0	5.2	4.9
				3	4.8	4.0	4.9	5.0	5.0	5.0	5.5	5.0
				5	4.7	5.1	5.2	5.1	5.1	5.1	3.8	5.0

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
0.5	25%	20	3	1	4.9	4.1	5.1	4.6	4.6	4.6	4.8	4.7
				2	5.2	5.0	5.4	4.9	4.9	4.9	4.3	4.9
			5	1	5.8	3.7	5.8	5.2	5.1	5.0	5.0	5.1
				2	5.6	3.9	5.5	4.7	4.7	4.7	5.0	4.9
				3	5.3	4.7	5.4	4.5	4.4	4.4	4.6	4.6
			7	1	6.6	3.8	6.3	5.1	5.0	4.8	5.0	5.0
				3	6.7	4.1	6.3	5.0	4.9	4.8	5.7	5.2
				5	6.5	5.5	6.2	4.9	4.8	4.8	3.1	5.1
		30	3	1	5.2	4.5	5.2	5.0	5.0	5.0	5.2	5.0
				2	5.0	5.0	5.3	4.9	4.9	4.9	4.6	4.8
			5	1	5.5	4.0	5.5	5.1	5.1	5.1	5.1	5.1
				2	5.3	4.3	5.5	5.0	4.9	4.9	5.3	5.1
				3	5.5	4.9	5.6	5.1	5.1	5.1	5.0	5.1
			7	1	5.6	3.7	5.6	5.0	5.0	4.9	5.1	4.9
				3	5.7	4.0	5.8	5.0	4.9	4.9	5.2	5.0
				5	5.7	5.2	5.6	4.9	4.8	4.8	3.4	4.8
		40	3	1	5.0	4.5	5.1	4.9	4.9	4.9	5.0	5.0
				2	5.2	5.1	5.5	5.1	5.1	5.1	4.9	5.2
			5	1	5.1	4.0	5.1	4.9	4.9	4.9	4.9	4.9
				2	5.0	4.1	5.2	4.9	4.9	4.9	4.9	4.9
				3	5.0	4.6	5.3	5.0	5.0	5.0	4.7	4.8
			7	1	5.2	3.6	5.3	4.9	4.9	4.9	5.0	4.9
				3	5.4	4.2	5.5	5.1	5.0	5.0	5.3	5.0
				5	5.2	5.3	5.5	4.8	4.8	4.8	4.4	4.8
	50%	20	3	1	6.0	4.7	6.0	5.0	4.9	4.8	5.1	5.1
				2	6.6	5.3	6.7	5.3	5.2	5.2	5.0	5.7
			5	1	6.8	4.7	7.0	5.3	5.2	5.0	4.8	5.6
				2	7.2	4.9	7.6	5.2	5.1	4.9	5.3	5.8
				3	7.3	5.3	7.7	5.1	4.9	4.8	5.2	5.8
			7	1	7.8	4.5	7.5	5.7	5.5	5.0	4.1	6.1
				3	8.8	5.5	9.2	5.6	5.4	4.9	5.7	6.2
				5	9.3	7.4	10.1	5.5	5.3	5.0	5.8	6.7
		30	3	1	5.6	4.5	5.5	4.9	4.9	4.9	4.9	4.9
				2	5.5	5.0	5.9	4.9	4.9	4.8	4.9	5.1
			5	1	6.2	4.5	6.2	5.2	5.1	5.0	4.9	5.1
				2	6.2	4.7	6.6	4.9	4.8	4.8	5.2	5.3
				3	6.5	5.5	6.8	4.8	4.8	4.7	5.6	5.5
			7	1	6.3	4.4	6.4	4.9	4.8	4.6	4.7	5.1
				3	7.4	5.1	7.9	5.0	5.0	4.8	5.6	5.7
				5	7.4	6.0	8.1	5.0	5.0	4.8	5.3	5.9
		40	3	1	5.6	4.8	5.7	5.0	5.0	5.0	5.1	5.1
				2	5.5	5.1	5.8	4.9	4.9	4.9	5.0	5.1
			5	1	5.8	4.6	5.9	5.0	4.9	4.9	5.0	5.1
				2	5.9	4.6	6.2	5.0	4.9	4.9	5.0	5.2
				3	6.0	5.2	6.4	4.9	4.8	4.8	5.2	5.2
			7	1	6.5	5.0	6.6	5.3	5.3	5.2	5.4	5.5
				3	6.9	5.2	7.2	5.1	5.1	5.0	5.7	5.7
				5	6.7	5.8	7.4	5.1	5.0	5.0	5.3	5.8

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	10%	20	3	1	5.2	4.2	5.1	4.9	4.9	4.9	4.9	4.7
				2	5.0	4.7	5.0	4.6	4.6	4.6	4.2	4.4
			5	1	5.6	3.6	5.4	5.0	4.9	4.9	5.0	4.7
				2	5.1	3.7	5.0	4.5	4.5	4.5	4.8	4.3
				3	5.4	4.6	5.3	4.7	4.7	4.7	4.6	4.4
			7	1	5.7	3.2	5.7	4.5	4.4	4.3	4.6	4.3
				3	5.9	3.4	5.3	4.4	4.4	4.3	4.9	4.2
				5	6.0	4.5	5.3	4.8	4.7	4.7	2.2	4.0
		30	3	1	5.2	4.3	5.2	5.1	5.1	5.1	4.9	5.0
				2	5.0	4.9	5.2	5.0	5.0	5.0	4.6	4.8
			5	1	5.0	3.8	5.0	4.7	4.7	4.7	4.9	4.6
				2	5.1	4.2	5.2	4.8	4.8	4.8	5.1	4.8
				3	5.0	4.5	5.1	4.7	4.7	4.7	4.6	4.5
			7	1	5.5	3.6	5.5	5.1	5.0	5.0	5.1	4.9
				3	5.3	4.0	5.2	4.7	4.7	4.7	5.1	4.6
				5	5.4	5.0	5.2	4.8	4.8	4.8	3.8	4.3
		40	3	1	4.8	4.3	4.9	4.8	4.8	4.8	4.8	4.7
				2	5.0	5.2	5.1	5.0	5.0	5.0	5.0	4.8
			5	1	5.1	4.1	5.3	4.9	4.9	4.9	5.1	5.0
				2	5.0	4.4	5.2	4.9	4.9	4.9	5.0	4.8
				3	4.7	4.5	4.9	4.6	4.6	4.6	4.6	4.4
			7	1	5.1	3.7	5.1	4.8	4.8	4.8	5.1	4.8
				3	5.1	4.1	5.1	4.7	4.7	4.7	5.0	4.6
				5	4.9	5.0	5.0	4.6	4.6	4.6	4.2	4.2
	25%	20	3	1	5.7	4.5	5.5	4.8	4.7	4.7	4.8	4.5
				2	6.1	4.7	5.7	5.1	5.1	5.1	4.6	4.7
			5	1	6.6	4.5	6.4	5.0	4.9	4.9	4.9	4.6
				2	7.1	4.5	6.7	5.2	5.0	4.9	5.0	4.7
				3	7.0	4.9	6.5	5.1	5.0	4.9	4.8	4.6
			7	1	7.2	4.8	7.3	5.4	5.2	5.0	4.8	4.3
				3	8.1	4.3	7.4	4.9	4.7	4.5	4.9	4.3
				5	8.4	4.6	7.2	5.2	5.0	4.9	3.9	4.3
		30	3	1	5.4	4.8	5.4	4.9	4.8	4.8	5.1	4.8
				2	5.6	4.8	5.4	5.0	4.9	4.9	4.6	4.6
			5	1	5.8	4.6	5.9	4.9	4.8	4.8	5.0	4.6
				2	6.0	4.4	6.0	4.9	4.9	4.8	4.7	4.6
				3	6.1	4.6	6.0	5.0	5.0	4.9	4.6	4.7
			7	1	6.3	4.6	6.5	5.2	5.1	5.0	5.1	4.7
				3	6.8	4.4	6.6	4.9	4.8	4.7	5.0	4.4
				5	7.0	4.8	6.6	5.1	5.0	4.9	4.3	4.4
		40	3	1	5.3	4.7	5.2	4.9	4.9	4.9	4.9	4.8
				2	5.6	5.1	5.4	5.1	5.1	5.1	5.1	4.8
			5	1	5.8	4.9	5.9	5.2	5.2	5.1	5.2	5.0
				2	5.8	4.8	5.8	5.0	5.0	4.9	5.0	4.8
				3	5.8	4.7	5.6	5.0	5.0	4.9	4.7	4.6
			7	1	5.7	4.6	5.8	4.7	4.7	4.6	5.0	4.4
				3	5.9	4.0	5.8	4.6	4.6	4.5	4.5	4.4
				5	6.4	4.8	6.1	5.0	5.0	4.9	4.4	4.4

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$. (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
1	50%	20	3	1	6.9	5.4	6.9	5.5	5.4	5.3	5.0	5.4
				2	7.0	4.8	7.0	5.3	5.2	5.1	4.9	5.5
			5	1	7.4	5.5	7.7	5.4	5.2	5.0	4.7	5.2
				2	8.5	5.6	8.7	5.8	5.5	5.2	5.3	5.9
				3	8.7	5.7	8.9	5.5	5.3	5.1	5.8	6.0
			7	1	9.4	6.3	9.4	6.2	5.8	5.3	4.5	5.9
				3	11.0	6.6	11.3	6.3	5.9	5.4	5.9	6.2
				5	11.3	6.7	11.1	6.1	5.8	5.3	6.9	5.9
		30	3	1	6.2	5.3	6.4	5.3	5.2	5.2	5.1	5.5
				2	6.4	5.2	6.7	5.2	5.2	5.1	5.2	5.6
			5	1	7.0	6.0	7.3	5.5	5.4	5.2	5.3	5.4
				2	7.5	5.6	7.6	5.4	5.3	5.1	5.2	5.6
				3	7.5	5.4	7.8	5.3	5.2	5.0	5.4	5.7
			7	1	7.5	5.8	7.8	5.4	5.3	5.1	4.8	5.2
				3	8.9	5.8	9.2	5.7	5.5	5.3	5.4	5.9
				5	9.1	5.6	9.3	5.6	5.4	5.2	5.7	6.0
		40	3	1	6.1	5.2	6.1	5.3	5.3	5.3	5.1	5.4
				2	6.0	5.1	6.1	5.0	5.0	4.9	5.1	5.2
			5	1	6.9	5.8	6.9	5.5	5.4	5.4	5.3	5.5
				2	6.9	5.5	7.2	5.4	5.3	5.2	5.2	5.5
				3	6.5	5.0	6.9	5.0	4.9	4.9	4.9	5.3
			7	1	6.7	5.6	7.0	5.2	5.1	5.0	5.0	5.1
				3	7.8	5.4	8.1	5.2	5.1	5.0	5.1	5.4
				5	8.0	5.0	8.2	5.4	5.2	5.1	5.0	5.8
3	10%	20	3	1	5.9	4.5	5.8	5.2	5.2	5.1	4.7	4.6
				2	5.8	4.5	5.2	4.8	4.7	4.7	4.5	3.9
			5	1	6.3	4.3	6.2	5.0	4.9	4.8	4.6	4.0
				2	6.6	4.2	5.9	5.0	4.9	4.8	4.6	3.8
				3	6.3	4.1	5.3	4.7	4.6	4.5	4.1	3.4
			7	1	6.8	4.4	6.7	4.8	4.7	4.5	4.5	3.1
				3	7.4	3.8	6.2	4.6	4.4	4.3	4.3	3.2
				5	7.7	4.0	5.4	4.7	4.5	4.4	3.6	2.6
		30	3	1	5.7	4.8	5.6	5.2	5.2	5.2	5.0	4.9
				2	5.6	4.6	5.3	4.9	4.9	4.9	4.6	4.4
			5	1	5.8	4.5	5.7	4.8	4.8	4.8	4.8	4.4
				2	5.8	4.4	5.6	4.7	4.7	4.6	4.6	3.9
				3	5.8	4.2	5.2	4.7	4.6	4.6	4.2	3.7
			7	1	6.3	4.6	6.4	5.1	5.0	4.9	4.9	4.1
				3	6.5	4.1	5.9	4.7	4.6	4.6	4.4	3.7
				5	6.8	4.0	5.3	4.8	4.8	4.7	3.7	3.4
		40	3	1	5.2	4.7	5.1	4.7	4.7	4.7	4.9	4.5
				2	5.3	4.6	5.0	4.8	4.8	4.8	4.6	4.3
			5	1	5.4	4.5	5.4	4.9	4.8	4.8	4.7	4.5
				2	5.6	4.6	5.4	4.8	4.7	4.7	4.7	4.3
				3	6.0	4.8	5.4	4.9	4.9	4.9	4.8	4.3
			7	1	5.8	4.6	5.8	4.9	4.8	4.8	4.9	4.3
				3	6.4	4.7	6.0	5.0	5.0	4.9	4.9	4.2
				5	6.4	4.5	5.5	5.0	5.0	4.9	4.1	3.7

Simulated rejection rates for $H_0 : \boldsymbol{\beta}_q = 0_q$ (Continuation).

σ	C	n	p	q	SLR	SR	ST	SLR*1	SLR*2	SLR*3	SR*	ST*
3	25%	20	3	1	7.0	5.4	6.8	5.8	5.7	5.7	5.1	5.0
				2	6.6	3.9	5.7	5.1	5.1	5.0	3.9	4.1
			5	1	7.7	5.6	7.5	5.5	5.4	5.3	4.8	4.2
				2	8.1	4.6	7.3	5.6	5.4	5.2	4.3	4.2
				3	8.2	3.8	6.7	5.4	5.3	5.1	3.7	4.1
			7	1	8.4	6.1	8.4	5.8	5.6	5.3	4.7	3.5
				3	10.2	4.5	8.8	5.8	5.4	5.0	4.0	3.7
				5	10.8	3.1	7.6	5.8	5.5	5.1	3.1	3.2
		30	3	1	5.9	5.0	5.8	5.1	5.0	5.0	4.8	4.6
				2	6.2	4.3	5.8	5.2	5.2	5.1	4.3	4.6
			5	1	6.8	5.7	6.8	5.4	5.3	5.2	5.0	4.7
				2	7.0	4.9	6.6	5.3	5.2	5.1	4.6	4.5
				3	7.3	4.1	6.4	5.4	5.3	5.2	3.9	4.2
			7	1	7.4	5.9	7.5	5.5	5.4	5.2	5.0	4.1
				3	8.5	4.7	7.7	5.4	5.2	5.1	4.2	4.1
				5	8.7	3.5	6.9	5.6	5.5	5.3	3.5	3.7
		40	3	1	5.8	5.1	5.8	5.1	5.1	5.1	4.9	4.9
				2	5.9	4.5	5.6	5.1	5.1	5.1	4.5	4.7
			5	1	6.5	5.4	6.5	5.3	5.3	5.2	5.0	4.9
				2	6.7	5.0	6.5	5.3	5.3	5.2	4.6	4.7
				3	6.7	4.4	6.1	5.2	5.2	5.1	4.2	4.5
			7	1	7.2	6.0	7.3	5.6	5.5	5.5	5.3	4.8
				3	7.5	4.8	7.2	5.4	5.3	5.1	4.4	4.5
				5	7.9	3.9	6.7	5.5	5.4	5.4	3.9	4.0
	50%	20	3	1	7.2	5.8	7.4	5.3	5.2	5.0	5.0	5.6
				2	7.5	4.6	7.7	5.5	5.4	5.2	4.7	5.9
			5	1	9.0	6.9	9.1	6.2	5.8	5.5	5.3	6.0
				2	9.0	5.8	9.3	5.7	5.5	5.2	4.8	5.7
				3	10.0	4.8	10.1	6.0	5.7	5.4	4.6	5.9
			7	1	10.7	8.2	10.8	6.9	6.4	5.8	5.5	5.8
				3	12.8	6.7	12.7	6.9	6.3	5.6	5.3	5.8
				5	13.3	4.5	12.3	6.7	6.2	5.6	4.8	5.6
		30	3	1	6.5	5.5	6.7	5.1	5.1	5.0	5.0	5.5
				2	6.6	4.8	6.9	5.3	5.2	5.2	4.8	5.5
			5	1	7.5	6.3	7.8	5.4	5.3	5.1	5.2	5.5
				2	8.2	6.0	8.5	5.6	5.5	5.3	5.1	5.9
				3	8.2	5.0	8.2	5.6	5.4	5.2	4.9	5.9
			7	1	8.4	7.2	8.8	5.9	5.6	5.3	5.4	5.6
				3	10.1	6.3	10.3	6.1	5.7	5.4	5.1	6.0
				5	10.3	4.7	10.4	6.1	5.8	5.5	4.8	6.0
		40	3	1	6.1	5.5	6.2	5.2	5.1	5.1	5.2	5.3
				2	6.1	4.9	6.4	5.0	5.0	4.9	4.9	5.4
			5	1	7.0	6.1	7.1	5.4	5.3	5.2	5.2	5.6
				2	7.3	5.7	7.5	5.3	5.2	5.1	5.1	5.6
				3	7.0	4.8	7.4	5.1	5.0	4.9	4.7	5.5
			7	1	7.4	6.4	7.5	5.5	5.3	5.1	5.3	5.5
				3	8.3	6.0	8.8	5.5	5.4	5.1	5.2	5.9
				5	8.9	4.7	9.1	5.7	5.5	5.3	4.8	6.1

C Cumulants

Let $Y_1,\ldots,Y_n$ a random sample of a censored data from a Weibull distribution, the logarithm of the likelihood function is given by

$$\ell(\boldsymbol{\beta})=\sum_{i=1}^{n}\left\{\delta_i\left[-n\log\sigma+\frac{y_i-\mu_i}{\sigma}\right]-\exp\left(\frac{y_i-\mu_i}{\sigma}\right)\right\}. \tag{1}$$

The first four derivatives of (1) can be expressed, respectively, as

$$\begin{aligned}
\frac{\partial}{\partial\beta_r}\ell(\boldsymbol{\beta})&=\frac{1}{\sigma}\sum_{i=1}^{n}\left\{-\delta_i+\exp\left(\frac{y_i-\mu_i}{\sigma}\right)\right\}x_{ri};\\
\frac{\partial}{\partial\beta_r\partial\beta_s}\ell(\boldsymbol{\beta})&=-\frac{1}{\sigma^2}\sum_{i=1}^{n}\exp\left(\frac{y_i-\mu_i}{\sigma}\right)x_{ri}x_{si};\\
\frac{\partial}{\partial\beta_r\partial\beta_s\partial\beta_t}\ell(\boldsymbol{\beta})&=\frac{1}{\sigma^3}\sum_{i=1}^{n}\exp\left(\frac{y_i-\mu_i}{\sigma}\right)x_{ri}x_{si}x_{ti};\\
\frac{\partial}{\partial\beta_r\partial\beta_s\partial\beta_t\partial\beta_u}\ell(\boldsymbol{\beta})&=-\frac{1}{\sigma^4}\sum_{i=1}^{n}\exp\left(\frac{y_i-\mu_i}{\sigma}\right)x_{ri}x_{si}x_{ti}x_{ui}.
\end{aligned}$$

The second- to forth-oder cumulants are:

$$\begin{aligned}
&\kappa_{rs}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}w_ix_{ri}x_{si};\;\;\kappa_{r,s}=-\kappa_{rs}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}w_ix_{ri}x_{si};\\
&\kappa_{rst}=\frac{1}{\sigma^3}\sum_{i=1}^{n}w_ix_{ri}x_{si}x_{ti};\;\;\kappa_{rs}^{(t)}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}w_i'x_{ri}x_{si}x_{ti};\\
&\kappa_{rs,t}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(w_i'+\frac{1}{\sigma}w_i\right)x_{ri}x_{si}x_{ti};\;\;\kappa_{r,s,t}=\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(3w_i'+\frac{2}{\sigma}w_i\right)x_{ri}x_{si}x_{ti};\\
&\kappa_{rstu}=-\frac{1}{\sigma^4}\sum_{i=1}^{n}w_ix_{ri}x_{si}x_{ti}x_{ui};\\
&\kappa_{rs}^{(tu)}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}w_i''x_{ri}x_{si}x_{ti}x_{ui};\;\;\kappa_{rst}^{(u)}=\frac{1}{\sigma^3}\sum_{i=1}^{n}w_i'x_{ai}x_{ri}x_{si}x_{ti}x_{ui};\\
&\kappa_{r,stu}=\frac{1}{\sigma^3}\sum_{i=1}^{n}\left(\frac{1}{\sigma}w_i+w_i'\right)x_{ri}x_{si}x_{ti}x_{ui};\;\;\kappa_{rs,tu}=\frac{1}{\sigma^4}\sum_{i=1}^{n}(2w_i+2\sigma w_i'-w_i^2)x_{ri}x_{si}x_{ti}x_{ui};\\
&\kappa_{r,s,tu}=\frac{1}{\sigma^2}\sum_{i=1}^{n}\left\{\frac{1}{\sigma^2}w_i^2-\frac{3}{\sigma^2}w_i-\frac{4}{\sigma}w_i'-w_i''\right\}x_{ri}x_{si}x_{ti}x_{ui};\\
&\kappa_{r,s,t,u}=\frac{1}{\sigma^2}\sum_{i=1}^{n}\left\{-\frac{3}{\sigma^2}w_i^2+\frac{9}{\sigma^2}w_i+\frac{14}{\sigma}w_i'+6w_i''\right\}x_{ri}x_{si}x_{ti}x_{ui};
\end{aligned}$$

where

$$\begin{aligned}
&w_i=1-\exp\left\{-L_i^{1/\sigma}\exp(-\mu_i/\sigma)\right\},\;w_i'=-\frac{1}{\sigma}L_i^{1/\sigma}\exp\{-L_i^{1/\sigma}\exp(-\mu_i/\sigma)-\mu_i/\sigma\},\\
&w_i''=-\frac{1}{\sigma^2}L_i^{1/\sigma}\exp\{-L_i^{1/\sigma}\exp(-\mu_i/\sigma)-\mu_i/\sigma\}\left[L_i^{1/\sigma}\exp(-\mu_i/\sigma)-1\right].
\end{aligned}$$

It can be observed that $w_i'=w_i''=0$ for type II censoring.

D Details about corrections

D.1 Likelihood ratio test

From the result of Lawley (1956), the Bartlett-correction factor for LR statistic for testing $\mathcal{H}: \boldsymbol{\beta}_1 = \boldsymbol{\beta}_1^{(0)}$ is

$$\varepsilon_p = \sum (l_{rstu} - l_{rstuvw}), \tag{2}$$

where

$$l_{rstu} = \kappa^{rs}\kappa^{tu}\left\{\frac{1}{4}\kappa_{rstu} - \kappa_{rst}^{(u)} + \kappa_{rt}^{(su)}\right\},$$

$$l_{rstuvw} = \kappa^{rs}\kappa^{tu}\kappa^{vw}\left\{\kappa_{rtv}\left(\frac{1}{6}\kappa_{suw} - \kappa_{sw}^{(u)}\right) + \kappa_{rt}^{(v)}\kappa_{sw}^{(u)} + \kappa_{rtu}\left(\frac{1}{4}\kappa_{svw} - \kappa_{sw}^{(v)}\right) + \kappa_{rt}^{(u)}\kappa_{sw}^{(v)}\right\}.$$

In censored data from a Weibull distribution, equation (2) is

$$l_{rstu} = \frac{1}{4\sigma^2}\sum_{i=1}^{n} z_{ii}^2 f_{1i},$$

where $f_{1i} = -\sigma^{-2}w_i - 4\sigma^{-1}w_i' - 4w_i''$ and $z_{ii} = -\sum_{r,s} x_{ri}\kappa^{rs}x_{si} = -\sum_{t,u} x_{ti}\kappa^{tu}x_{ui}$,

$$l_{rstuvw} = -\frac{1}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i\left(\frac{1}{6\sigma}w_j + w_j'\right)z_{ij}^3 - \frac{1}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i'w_j'z_{ij}^3$$
$$-\frac{1}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i\left(\frac{1}{4\sigma}w_j + w_j'\right)z_{ij}z_{ii}z_{jj} - \frac{1}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i'w_j'z_{ij}z_{ii}z_{jj},$$

where $z_{ij} = -\sum_{r,s} x_{ri}\kappa^{rs}x_{sj} = -\sum_{t,u} x_{ti}\kappa^{tu}x_{uj} = -\sum_{v,w} x_{vi}\kappa^{vw}x_{wj}$. In matrix notation

$$l_{rstu} = \frac{1}{4\sigma^2}\mathrm{tr}\left\{\mathbf{F}_1\dot{\mathbf{Z}}^{(2)}\right\},$$

$$l_{rstuvw} = -\sigma^{-5}\mathbf{1}^\top\mathbf{W}\mathbf{Z}^{(3)}\left(\frac{1}{6\sigma}\mathbf{W} + \mathbf{W}'\right)\mathbf{1} - \sigma^{-4}\mathbf{1}^\top\mathbf{W}'\mathbf{Z}^{(3)}\mathbf{W}'\mathbf{1}$$
$$-\sigma^{-5}\mathbf{1}^\top\mathbf{W}\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\left(\frac{1}{4\sigma}\mathbf{W} + \mathbf{W}'\right)\mathbf{1} - \sigma^{-4}\mathbf{1}^\top\mathbf{W}'\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\mathbf{W}'\mathbf{1},$$

where $\mathbf{Z} = \mathbf{X}\mathbf{K}^{-1}\mathbf{X}^\top$, $\mathbf{F}_1 = \mathrm{diag}\{f_{11}, \ldots, f_{1n}\}$, $\mathbf{W} = \mathrm{diag}\{w_1, \ldots, w_n\}$, $\mathbf{W}' = \mathrm{diag}\{w_1', \ldots, w_n'\}$, $\dot{\mathbf{Z}} = \mathrm{diag}\{z_{11}, \ldots, z_{nn}\}$, is a diagonal matrix with the elements of $\mathbf{Z}$, $\mathbf{Z}^{(2)} = \mathbf{Z}\odot\mathbf{Z}$, $\mathbf{Z}^{(3)} = \mathbf{Z}^{(2)}\odot\mathbf{Z}$, $\odot$ represents a direct product and $\mathbf{1}$ is a vector of ones. For convenience, we can write l_{rstuvw} as

$$l_{rstuvw} = -(1/6)\sigma^{-6}\mathbf{1}^\top\mathbf{W}\mathbf{Z}^{(3)}\mathbf{W}\mathbf{1} - \sigma^{-4}\mathbf{1}^\top\left(\sigma^{-1}\mathbf{W} + \mathbf{W}'\right)\mathbf{Z}^{(3)}\mathbf{W}'\mathbf{1}$$
$$-(1/4)\sigma^{-5}\mathbf{1}^\top\mathbf{W}\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\left(\sigma^{-1}\mathbf{W} + 4\mathbf{W}'\right)\mathbf{1} - \sigma^{-4}\mathbf{1}^\top\mathbf{W}'\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\mathbf{W}'\mathbf{1},$$

Finally,

$$\varepsilon_p = (1/4)\sigma^{-2}\mathrm{tr}\left\{\mathbf{F}_1\dot{\mathbf{Z}}^{(2)}\right\} + (1/12)\sigma^{-6}\mathbf{1}^\top\mathbf{W}\left(2\mathbf{Z}^{(3)} + 3\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\right)\mathbf{W}\mathbf{1}$$
$$+\sigma^{-5}\mathbf{1}^\top\mathbf{W}\left(\mathbf{Z}^{(3)} + \dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\right)\mathbf{W}'\mathbf{1} + \sigma^{-4}\mathbf{1}^\top\mathbf{W}'\left(\mathbf{Z}^{(3)} + \dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}}\right)W'\mathbf{1}.$$

D.2 Score test

Suppose the hypothesis test $\mathcal{H}: \boldsymbol{\beta}_1 = \boldsymbol{\beta}_1^{(0)}$ and the partition of the parameter vector $\boldsymbol{\beta} = (\boldsymbol{\beta}_1^\top, \boldsymbol{\beta}_2^\top)^\top$. This partition induces the corresponding partitions

$$\mathbf{K}_{\boldsymbol{\beta}\boldsymbol{\beta}} = \begin{pmatrix} \mathbf{K}_{\boldsymbol{\beta}_1\boldsymbol{\beta}_1} & \mathbf{K}_{\boldsymbol{\beta}_1\boldsymbol{\beta}_2} \\ \mathbf{K}_{\boldsymbol{\beta}_2\boldsymbol{\beta}_1} & \mathbf{K}_{\boldsymbol{\beta}_2\boldsymbol{\beta}_2} \end{pmatrix}, \ \mathbf{K}_{\boldsymbol{\beta}\boldsymbol{\beta}}^{-1} = \begin{pmatrix} \mathbf{K}_{\boldsymbol{\beta}_1\boldsymbol{\beta}_1}^{-1} & \mathbf{K}_{\boldsymbol{\beta}_1\boldsymbol{\beta}_2}^{-1} \\ \mathbf{K}_{\boldsymbol{\beta}_2\boldsymbol{\beta}_1}^{-1} & \mathbf{K}_{\boldsymbol{\beta}_2\boldsymbol{\beta}_2}^{-1} \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\boldsymbol{\beta}_2\boldsymbol{\beta}_2}^{-1} \end{pmatrix} \text{ and } \mathbf{M} = \mathbf{K}_{\boldsymbol{\beta}\boldsymbol{\beta}}^{-1} - \mathbf{A}.$$

From Cordeiro and Ferrari (1991), the Bartlett-type corrected score statistic is given by

$$S_R^* = S_R\{1 - \left(c_R + b_R S_R + a_R S_R^2\right)\}, \tag{3}$$

where

$$a_R = \frac{A_{R3}}{12q(q+2)(q+4)}, \ \ b_R = \frac{A_{R2} - 2A_{R3}}{12q(q+2)}, \ \ c_R = \frac{A_{R1} - A_{R2} + A_{R3}}{12q},$$

$$\begin{aligned}
A_{R1} &= 3\sum(\kappa_{rst} + 2\kappa_{r,st})(\kappa_{uvw} + 2\kappa_{uv,w})a^{rs}a^{vw}m^{tu} - 6\sum(\kappa_{rst} + 2\kappa_{r,st})\kappa_{u,v,w}a^{rs}a^{tu}m^{vw} \\
&\quad + 6\sum(\kappa_{r,st} - \kappa_{r,s,t})(\kappa_{uvw} + 2\kappa_{uv,w})a^{sv}a^{tw}m^{ru} - 6\sum(\kappa_{r,s,t,u} + \kappa_{r,s,tu})a^{tu}m^{rs}, \\
A_{R2} &= -3\sum\kappa_{r,s,t}\kappa_{u,v,w}a^{tu}m^{rs}m^{vw} + 6\sum(\kappa_{rst} + 2\kappa_{r,st})\kappa_{u,v,w}a^{rs}m^{tu}m^{vw} \\
&\quad - 6\sum\kappa_{r,s,t}\kappa_{u,v,w}a^{tw}m^{ru}m^{sv} + 3\sum\kappa_{r,s,t,u}m^{rs}m^{tu}, \\
A_{R3} &= 3\sum\kappa_{r,s,t}\kappa_{u,v,w}m^{rs}m^{tu}m^{vw} + 2\sum\kappa_{r,s,t}\kappa_{u,v,w}m^{ru}m^{sv}m^{tw}.
\end{aligned}$$

In censored data from a Weibull distribution, equation (3) is

$$\begin{aligned}
A_{R1} &= \frac{3}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{\sigma}w_i + 2w_i'\right)\left(\frac{1}{\sigma}w_j + 2w_j'\right)z_{p-q,ii}z_{p-q,jj}(z_{p,ij} - z_{p-q,ij}) \\
&\quad + \frac{6}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{\sigma}w_i + 2w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)z_{p-q,ii}z_{p-q,ij}(z_{p,jj} - z_{p-q,jj}) \\
&\quad + \frac{6}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{3}{\sigma}w_i + 4w_i'\right)\left(\frac{1}{\sigma}w_j + 2w_j'\right)z_{p-q,ij}z_{p-q,ij}(z_{p,ij} - z_{p-q,ij}) \\
&\quad - \frac{6}{\sigma^2}\sum_{i=1}^{n}\left(-\frac{2}{\sigma^2}w_i^2 + \frac{6}{\sigma^2}w_i + \frac{10}{\sigma}w_i' + 5w_i''\right)z_{p-q,ii}(z_{p,ii} - z_{p-q,ii}),
\end{aligned}$$

$$\begin{aligned}
A_{R2} &= -\frac{3}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{2}{\sigma}w_i + 3w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)z_{p-q,ij}(z_{p,ii} - z_{p-q,ii})(z_{p,jj} - z_{p-q,jj}) \\
&\quad - \frac{6}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{\sigma}w_i + 2w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)z_{p-q,ii}(z_{p,ij} - z_{p-q,ij})(z_{p,jj} - z_{p-q,jj}) \\
&\quad - \frac{6}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{2}{\sigma}w_i + 3w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)z_{p-q,ij}(z_{p,ij} - z_{p-q,ij})(z_{p,ij} - z_{p-q,ij}) \\
&\quad + \frac{3}{\sigma^2}\sum_{i=1}^{n}\left(-\frac{3}{\sigma^2}w_i^2 + \frac{9}{\sigma^2}w_i + \frac{14}{\sigma}w_i' + 6w_i''\right)(z_{p,ii} - z_{p-q,ii})(z_{p,ii} - z_{p-q,ii}),
\end{aligned}$$

$$A_{R3} = \frac{3}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{2}{\sigma}w_i + 3w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)(z_{p,ii} - z_{p-q,ii})(z_{p,ij} - z_{p-q,ij})(z_{p,jj} - z_{p-q,jj})$$
$$+ \frac{2}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{2}{\sigma}w_i + 3w_i'\right)\left(\frac{2}{\sigma}w_j + 3w_j'\right)(z_{p,ij} - z_{p-q,ij})(z_{p,ij} - z_{p-q,ij})(z_{p,ij} - z_{p-q,ij}).$$

In matrix notation

$$\begin{aligned}
A_{R1} = {} & 3\sigma^{-6}\mathbf{1}^\top\left(\mathbf{W} + 2\sigma\mathbf{W}'\right)\dot{\mathbf{Z}}_2\left(\mathbf{Z} - \mathbf{Z}_2\right)\dot{\mathbf{Z}}_2\left(\mathbf{W} + 2\sigma\mathbf{W}'\right)\mathbf{1} \\
& + 6\sigma^{-6}\mathbf{1}^\top\left(\mathbf{W} + 2\sigma\mathbf{W}'\right)\dot{\mathbf{Z}}_2\mathbf{Z}_2\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\mathbf{1} \\
& + 6\sigma^{-6}\mathbf{1}^\top\left(3\mathbf{W} + 4\sigma\mathbf{W}'\right)\left[\mathbf{Z}_2^{(2)} \odot \left(\mathbf{Z} - \mathbf{Z}_2\right)\right]\left(\mathbf{W} + 2\sigma\mathbf{W}'\right)\mathbf{1} \\
& - 6\sigma^{-2}\mathrm{tr}\left\{\mathbf{F}_2\dot{\mathbf{Z}}_2\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\right\},
\end{aligned}$$

$$\begin{aligned}
A_{R2} = {} & -3\sigma^{-6}\mathbf{1}^\top\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\mathbf{Z}_2\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\mathbf{1} \\
& - 6\sigma^{-6}\mathbf{1}^\top\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z} - \mathbf{Z}_2\right)\dot{\mathbf{Z}}_2\left(\mathbf{W} + 2\sigma\mathbf{W}'\right)\mathbf{1} \\
& - 6\sigma^{-6}\mathbf{1}^\top\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\left[\mathbf{Z}_2 \odot \left(\mathbf{Z} - \mathbf{Z}_2\right)^{(2)}\right]\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\mathbf{1} \\
& + 3\sigma^{-2}\mathrm{tr}\left\{\mathbf{F}_3\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)^{(2)}\right\},
\end{aligned}$$

$$A_{R3} = \sigma^{-6}\mathbf{1}^\top\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\left\{3\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z} - \mathbf{Z}_2\right)\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right) + 2\left(\mathbf{Z} - \mathbf{Z}_2\right)^{(3)}\right\}\left(2\mathbf{W} + 3\sigma\mathbf{W}'\right)\mathbf{1},$$

where $\mathbf{F}_2 = \mathrm{diag}\left\{-2\sigma^{-2}w_i^2 + 6\sigma^{-2}w_i + 10\sigma^{-1}w_i' + 5w_i''\right\}$ and $\mathbf{F}_3 = \mathrm{diag}\left\{-3\sigma^{-2}w_i^2 + 9\sigma^{-2}w_i + 14\sigma^{-1}w_i' + 6w_i''\right\}$.

D.3 Gradient test

From Vargas et al. (2013), the Bartlett-type corrected gradient statistic is

$$S_T^* = S_T\left\{1 - \left(c_T + b_T S_T + a_T S_T^2\right)\right\}, \tag{4}$$

where

$$a_T = \frac{A_{T3}}{12q(q+2)(q+4)}, \quad b_T = \frac{A_{T2} - 2A_{T3}}{12q(q+2)}, \quad c_T = \frac{A_{T1} - A_{T2} + A_{T3}}{12q},$$

$$\begin{aligned}
A_{T1} = {} & 3\sum \kappa_{rst}\kappa_{uvw}[m^{rs}a^{vw}(m^{tu} + 2a^{tu}) + a^{rs}m^{tu}a^{vw} + 2m^{ru}a^{sv}a^{tw}] \\
& - 12\sum \kappa_{rs}^{(t)}\kappa_{uv}^{(w)}(\kappa^{rt}\kappa^{su}\kappa^{vw} + a^{rt}a^{su}a^{vw} + \kappa^{tu}\kappa^{rv}\kappa^{sw} + a^{tu}a^{rv}a^{sw}) \\
& - 6\sum \kappa_{rst}\kappa_{uv}^{(w)}[(a^{tw} - \kappa^{tw})(\kappa^{ru}\kappa^{sv} - a^{ru}a^{sv}) + m^{rs}(a^{tu}a^{vw} + \kappa^{tu}\kappa^{vw}) \\
& + 2a^{st}(\kappa^{ru}\kappa^{vw} - a^{ru}a^{vw}) + 2a^{su}a^{tv}m^{rw}] \\
& + 6\sum \kappa_{rstu}m^{rs}a^{tu} - 6\sum \kappa_{rst}^{(u)}[m^{rs}(a^{tu} - \kappa^{tu}) + 2m^{ru}a^{st}] \\
& + 12\sum \kappa_{st}^{(ru)}(\kappa^{rs}\kappa^{tu} - a^{rs}a^{tu}),
\end{aligned}$$

$$A_{T2} = -3\sum \kappa_{rst}\kappa_{uvw}\left[\frac{1}{4}m^{rs}m^{tu}(4a^{vw}+3m^{wv}) + m^{rs}a^{tu}m^{vw} + \frac{1}{2}m^{ru}m^{sv}(4a^{tw}+m^{tw})\right]$$
$$+6\sum \kappa_{rst}\kappa_{uv}^{(w)}[m^{tw}(\kappa^{ru}\kappa^{sv} - a^{ru}a^{sv}) + m^{rs}(\kappa^{tu}\kappa^{vw} - a^{tu}a^{vw})]$$
$$+6\sum \kappa_{rst}^{(u)}m^{rs}m^{tu} - 3\sum \kappa_{rstu}m^{rs}m^{tu},$$
$$A_{T3} = \frac{1}{4}\sum \kappa_{rst}\kappa_{uvw}(3m^{rs}m^{tu}m^{vw} + 2m^{ru}m^{sv}m^{tw}),$$

Replacing equation (4) with the cumulants of censored data from a Weibull distribution,

$$A_{T1} = \frac{3}{\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j\left(z_{p,ii} - z_{p-q,ii}\right)z_{p-q,jj}\left[(z_{p,ij} - z_{p-q,ij}) + 2z_{p-q,ij}\right]$$
$$+\frac{3}{\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j z_{p-q,ii}\left(z_{p,ij} - z_{p-q,ij}\right)z_{p-q,jj}$$
$$+\frac{6}{\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j\left(z_{p,ij} - z_{p-q,ij}\right)z_{p-q,ij}z_{p-q,ij}$$
$$+\frac{12}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i' w_j' z_{p,ii}z_{p,ij}z_{p,jj}$$
$$-\frac{12}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i' w_j' z_{p-q,ii}z_{p-q,ij}z_{p-q,jj}$$
$$+\frac{12}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i' w_j' z_{p,ij}z_{p,ij}z_{p,ij}$$
$$-\frac{12}{\sigma^4}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i' w_j' z_{p-q,ij}z_{p-q,ij}z_{p-q,ij}$$
$$+\frac{6}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j'\left(z_{p-q,ij} + z_{p,ij}\right)\left(z_{p,ij}z_{p,ij} - z_{p-q,ij}z_{p-q,ij}\right)$$
$$+\frac{6}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j'\left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p-q,ij}z_{p-q,jj} + z_{p,ij}z_{p,jj}\right)$$
$$+\frac{12}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j' z_{p-q,ii}\left(z_{p,ij}z_{p,jj} - z_{p-q,ij}z_{p-q,jj}\right)$$
$$+\frac{12}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j' z_{p-q,ij}z_{p-q,ij}\left(z_{p,ij} - z_{p-q,ij}\right)$$
$$-\frac{6}{\sigma^4}\sum_{i=1}^{n} w_i\left(z_{p,ii} - z_{p-q,ii}\right)z_{p-q,ii}$$
$$-\frac{6}{\sigma^3}\sum_{i=1}^{n} w_i'\left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p-q,ii} + z_{p,ii}\right)$$
$$-\frac{12}{\sigma^3}\sum_{i=1}^{n} w_i'\left(z_{p,ii} - z_{p-q,ii}\right)z_{p-q,ii}$$
$$-\frac{12}{\sigma^2}\sum_{i=1}^{n} w_i''\left(z_{p,ii}z_{p,ii} - z_{p-q,ii}z_{p-q,ii}\right),$$

$$
\begin{aligned}
A_{T2} = & -\frac{3}{4\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p,ij} - z_{p-q,ij}\right)\left[4z_{p-q,jj} + 3\left(z_{p,jj} - z_{p-q,jj}\right)\right] \\
& - \frac{3}{\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \left(z_{p,ii} - z_{p-q,ii}\right) z_{p-q,ij}\left(z_{p,jj} - z_{p-q,jj}\right) \\
& - \frac{3}{2\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \left(z_{p,ij} - z_{p-q,ij}\right)\left(z_{p,ij} - z_{p-q,ij}\right)\left[4z_{p-q,ij} + \left(z_{p,ij} - z_{p-q,ij}\right)\right] \\
& - \frac{6}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j' \left(z_{p,ij} - z_{p-q,ij}\right)\left(z_{p,ij}z_{p,ij} - z_{p-q,ij}z_{p-q,ij}\right) \\
& - \frac{6}{\sigma^5}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j' \left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p,ij}z_{p,jj} - z_{p-q,ij}z_{p-q,jj}\right) \\
& + \frac{6}{\sigma^3}\sum_{i=1}^{n} w_i' \left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p,ii} - z_{p-q,ii}\right) \\
& + \frac{3}{\sigma^4}\sum_{i=1}^{n} w_i \left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p,ii} - z_{p-q,ii}\right), \\
A_{T3} = & \frac{3}{4\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \left(z_{p,ii} - z_{p-q,ii}\right)\left(z_{p,ij} - z_{p-q,ij}\right)\left(z_{p,jj} - z_{p-q,jj}\right) \\
& + \frac{2}{4\sigma^6}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \left(z_{p,ij} - z_{p-q,ij}\right)\left(z_{p,ij} - z_{p-q,ij}\right)\left(z_{p,ij} - z_{p-q,ij}\right).
\end{aligned}
$$

In matrix notation and after some algebra:

$$
\begin{aligned}
A_{T1} = & \, 3\sigma^{-6}\mathbf{1}^\top\mathbf{W}\left\{\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z} + \mathbf{Z}_2\right)\dot{\mathbf{Z}}_2 + \dot{\mathbf{Z}}_2\left(\mathbf{Z} - \mathbf{Z}_2\right)\dot{\mathbf{Z}}_2 + 2\left(\mathbf{Z} - \mathbf{Z}_2\right)\odot\mathbf{Z}_2^{(2)}\right\}\mathbf{W}\mathbf{1} \\
& + 6\sigma^{-5}\mathbf{1}^\top\mathbf{W}\left\{\left(\mathbf{Z}_2 + \mathbf{Z}\right)\odot\left(\mathbf{Z}^{(2)} - \mathbf{Z}_2^{(2)}\right) + \left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z}_2\dot{\mathbf{Z}}_2 + \mathbf{Z}\dot{\mathbf{Z}}\right)\right. \\
& \left. + \, 2\left[\dot{\mathbf{Z}}_2\left(\mathbf{Z}\dot{\mathbf{Z}} - \mathbf{Z}_2\dot{\mathbf{Z}}_2\right) + \mathbf{Z}_2^{(2)}\odot\left(\mathbf{Z} - \mathbf{Z}_2\right)\right]\right\}\mathbf{W}'\mathbf{1} \\
& + 12\sigma^{-4}\mathbf{1}^\top\mathbf{W}'\left\{\dot{\mathbf{Z}}\mathbf{Z}\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\mathbf{Z}_2\dot{\mathbf{Z}}_2 + \mathbf{Z}^{(3)} - \mathbf{Z}_2^{(3)}\right\}\mathbf{W}'\mathbf{1} \\
& - 6\sigma^{-4}\,\mathrm{tr}\left\{\mathbf{W}\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\dot{\mathbf{Z}}_2 + \sigma\mathbf{W}'\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\dot{\mathbf{Z}} + 3\dot{\mathbf{Z}}_2\right) + 2\sigma^2\mathbf{W}''\left(\dot{Z}_p^{(2)} - \dot{Z}_{p-q}^{(2)}\right)\right\}, \\
A_{T2} = & - 3\sigma^{-6}\mathbf{1}^\top\mathbf{W}\left\{(1/4)\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z} - \mathbf{Z}_2\right)\left(3\dot{\mathbf{Z}} + \dot{\mathbf{Z}}_2\right)\right. \\
& \left. + \left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\mathbf{Z}_2\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right) + (1/2)\left(\mathbf{Z} - \mathbf{Z}_2\right)^{(2)}\odot\left(\mathbf{Z} + 3\mathbf{Z}_2\right)\right\}\mathbf{W}\mathbf{1} \\
& - 6\sigma^{-5}\mathbf{1}^\top\mathbf{W}\left\{\left(\mathbf{Z} - \mathbf{Z}_2\right)\odot\left(\mathbf{Z}^{(2)} - \mathbf{Z}_2^{(2)}\right) + \left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z}\dot{\mathbf{Z}} - \mathbf{Z}_2\dot{\mathbf{Z}}_2\right)\right\}\mathbf{W}'\mathbf{1} \\
& + 3\sigma^{-4}\,\mathrm{tr}\left\{\mathbf{W}\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)^{(2)} + 2\sigma\mathbf{W}'\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)^{(2)}\right\}, \\
A_{T3} = & \, (1/4)\sigma^{-6}\mathbf{1}^\top\mathbf{W}\left\{3\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right)\left(\mathbf{Z} - \mathbf{Z}_2\right)\left(\dot{\mathbf{Z}} - \dot{\mathbf{Z}}_2\right) + 2\left(\mathbf{Z} - \mathbf{Z}_2\right)^{(3)}\right\}\mathbf{W}\mathbf{1}.
\end{aligned}
$$

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